The Positionality of the Electromagnetic and Gravitational Theory

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Abstract

The purpose of this paper is to discuss the time component of an electromagnetic field, and to have an advanced argument from the electromagnetic theory to the general theory of relativity.

The second purpose is to show that "the general theory of relativity" and "the Electromagnetic and Gravitational theory" leads to the similar formulas.

The special theory of relativity is based on the Lorentz transformations and two postulates. The Lorentz transformations are consisted of rotations in Minkowski space. Anti-de Sitter space appears to be rolling up a Minkowski space which has constant negative scalar curvature. Vice versa, tangent space of the anti-de Sitter space is a Minkowski space.

Therefore we stand in the position that the anti-de Sitter space is the space which we live in. Then we are on a same ground as the general theory of relativity.

Contents

1. Preliminaries

1.1 The matrix-vector, calculation and its image

We identified the four−dimensional vector $(ct, x, y, z)^{\mathrm{T}} \in \mathbb{R}^{1,3}$ (Minkowski space) with the Hermitian matrix.

$$\begin{bmatrix} \mathrm{c}t & \\ & \boldsymbol{r} \end{bmatrix} = \begin{pmatrix} \mathrm{c}t + x & y + iz \\ y - iz & \mathrm{c}t - x \end{pmatrix} \in \left\{ X \in M(2,\mathbb{C}) \,/\, X^* = X \right\}, \{\boldsymbol{r}\} = \left(\mathrm{c}t, x, y, z\right)^{\mathrm{T}} \in \mathbb{R}^{1,3} \tag{1}$$

Let us call this expression (1) of vector a matrix-vector.

Note that this expression has a product between two matrix-vectors with simple matrix calculation.

$$\begin{bmatrix} \mathrm{c}t & \\ & \boldsymbol{r} \end{bmatrix}\begin{bmatrix} \mathrm{c}t' & \\ & \boldsymbol{r}' \end{bmatrix} = \begin{bmatrix} \mathrm{c}t\mathrm{c}t' + \underline{\boldsymbol{r}\cdot\boldsymbol{r}'} & \\ & \mathrm{c}t\boldsymbol{r}' + \boldsymbol{r}\mathrm{c}t' - i(\underline{\underline{\boldsymbol{r}\times\boldsymbol{r}'}}) \end{bmatrix}. \tag{2}$$

The underlined sections of expression (2) means that a single-underlined $\underline{\boldsymbol{r}\cdot\boldsymbol{r}'}$ is scalar product and a double-underlined $\underline{\underline{\boldsymbol{r}\times\boldsymbol{r}'}}$ is vector product.

This expression (2) has two features, one is the functionality as matrix and the other is the simplicity as vector.

Furthermore these calculational images are as follows:

From expression (2), the matrix-vector expressed as

$$\begin{bmatrix} \text{Time component(one-dim.)} & \\ & \text{Space component(three-dim.)} \end{bmatrix}.$$

Hence, e.g. the images of the time component

$$ab + \boldsymbol{A}\bullet\boldsymbol{B} \quad \text{is} \quad \begin{bmatrix} a & \\ & \boldsymbol{A} \end{bmatrix}\begin{bmatrix} b & \\ & \boldsymbol{B} \end{bmatrix}.$$

The images of the space component

$$a\boldsymbol{B} + \boldsymbol{A}b - i\boldsymbol{A}\times\boldsymbol{B} \quad \text{is} \quad \begin{bmatrix} a & \\ & \boldsymbol{A} \end{bmatrix}\begin{bmatrix} b & \\ & \boldsymbol{B} \end{bmatrix} \text{and} \begin{bmatrix} a & \\ & \boldsymbol{A} \end{bmatrix}\begin{bmatrix} b & \\ & \boldsymbol{B} \end{bmatrix}$$

where, a, b, $\boldsymbol{A}$, $\boldsymbol{B}$ are arbitrary vectors. The arrows and bold arrow are their map transformations methods for the images of time component and space component.

This means that the scalar product and the vector product are not independent. Therefore there are closely connections under the matrix product.

1.2 The Lorentz form and the figure of the imaginary angle Θ

When a particle moves to the x-direction at the speed $\boldsymbol{v}_x = v$. The speed v is a scalar, then we have the following relation:

$$\begin{cases} \mathrm{c}t' = \gamma(\mathrm{c}t - \boldsymbol{\beta}_x x) \\ x' = \gamma(x - \boldsymbol{\beta}_x \mathrm{c}t) \\ y' = y \\ z' = z \end{cases}, \quad \gamma = \frac{u_0}{\mathrm{c}} = \frac{1}{\sqrt{1 - \left(\frac{v}{\mathrm{c}}\right)^2}} = \cosh\Theta, \quad \gamma\boldsymbol{\beta}_x = \frac{\boldsymbol{u}_x}{\mathrm{c}} = \frac{\frac{v}{\mathrm{c}}}{\sqrt{1 - \left(\frac{v}{\mathrm{c}}\right)^2}} = \sinh\Theta.$$

This Lorentz transformation has the representation of matrix-vector such as

$$\begin{pmatrix} ct'+x' & y'+iz' \\ y'-iz' & ct'-x' \end{pmatrix} = \begin{pmatrix} \gamma(1-\boldsymbol{\beta}_x)(ct+x) & y+iz \\ y-iz & \gamma(1+\boldsymbol{\beta}_x)(ct-x) \end{pmatrix}$$
$$= \begin{pmatrix} \gamma_+ - \gamma_- & 0 \\ 0 & \gamma_+ + \gamma_- \end{pmatrix} \begin{pmatrix} ct+x & y+iz \\ y-iz & ct-x \end{pmatrix} \begin{pmatrix} \gamma_+ - \gamma_- & 0 \\ 0 & \gamma_+ + \gamma_- \end{pmatrix}$$

where $\Upsilon_+ = \sqrt{(\Upsilon+1)/2} = \cosh(\Theta/2)$, $\Upsilon_- = \sqrt{(\Upsilon-1)/2} = \sinh(\Theta/2)$.

Therefore,

$$\begin{bmatrix} ct' & \\ & \boldsymbol{r}' \end{bmatrix} = \begin{bmatrix} \gamma_+ & \\ & -\boldsymbol{\gamma}_0 \end{bmatrix} \begin{bmatrix} ct & \\ & \boldsymbol{r} \end{bmatrix} \begin{bmatrix} \gamma_+ & \\ & -\boldsymbol{\gamma}_0 \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} ct & \\ & \boldsymbol{r} \end{bmatrix}^{-}, \boldsymbol{\gamma}_0 = \gamma_- \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

(Cf. signature "−"s of both upper sides correspond to the contravariance of vectors).

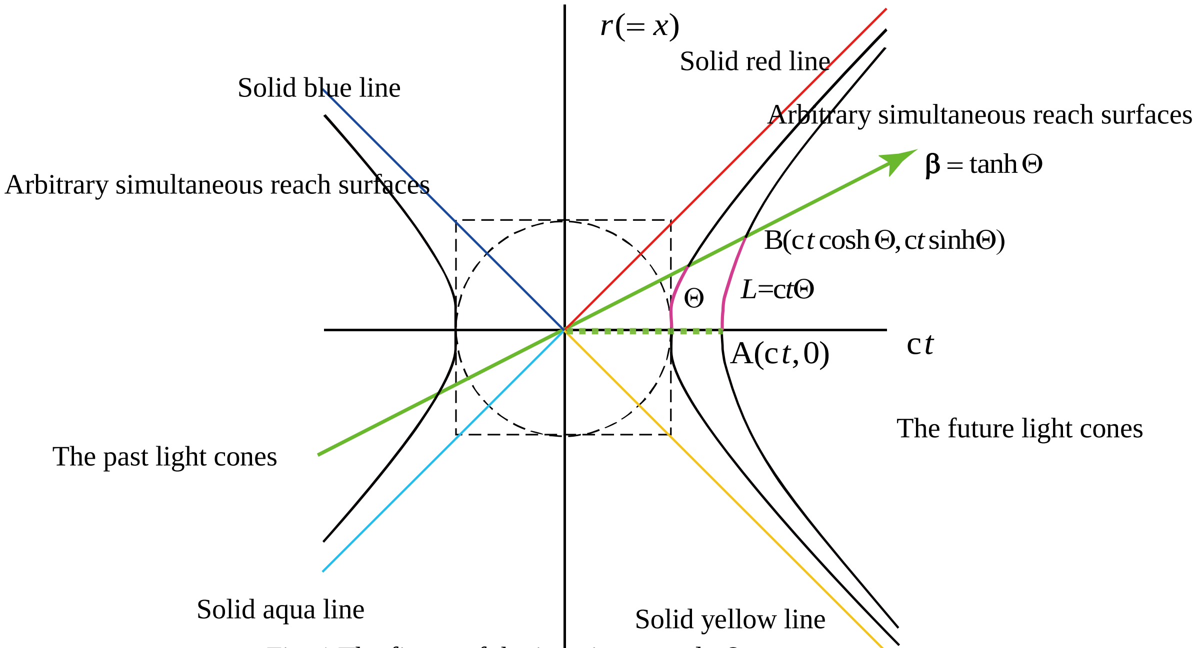

Fig. 1 The figure of the imaginary angle Θ.

Figure 1 shows a schematic of the figure of the imaginary angle Θ. The four null values in the Minkowski metric are shown in the four solid lines in Fig.1. The lines are called null lines. As to the figure of Θ, when the speed $\dfrac{dx}{dct} = \tanh\Theta$ is constant. Then by the formula $dct/d\tau = c\cosh\Theta$, $dx/d\tau = c\sinh\Theta$. We can put any point $B(ct\cosh\Theta, ct\sinh\Theta)$ by using the ct and Θ.

Then the arc length L on the hyperbolic line from point A(ct ,0) to point B is

$$L = \int_0^{\Theta} \sqrt{\left(\frac{dx}{d\Theta}\right)^2 - \left(\frac{dct}{d\Theta}\right)^2}\, d\Theta = ct\int_0^{\Theta} \sqrt{\cosh^2\Theta - \sinh^2\Theta}\, d\Theta = ct\int_0^{\Theta} d\Theta = ct\Theta.$$

(Cf. The arc length of the circle $(x,y) = r(\cos\theta, \sin\theta)$ is

$$L = \int_0^{\theta} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta = r\int_0^{\theta} \sqrt{\sin^2\theta + \cos^2\theta}\, d\theta = r\int_0^{\theta} d\theta = r\theta\,)$$

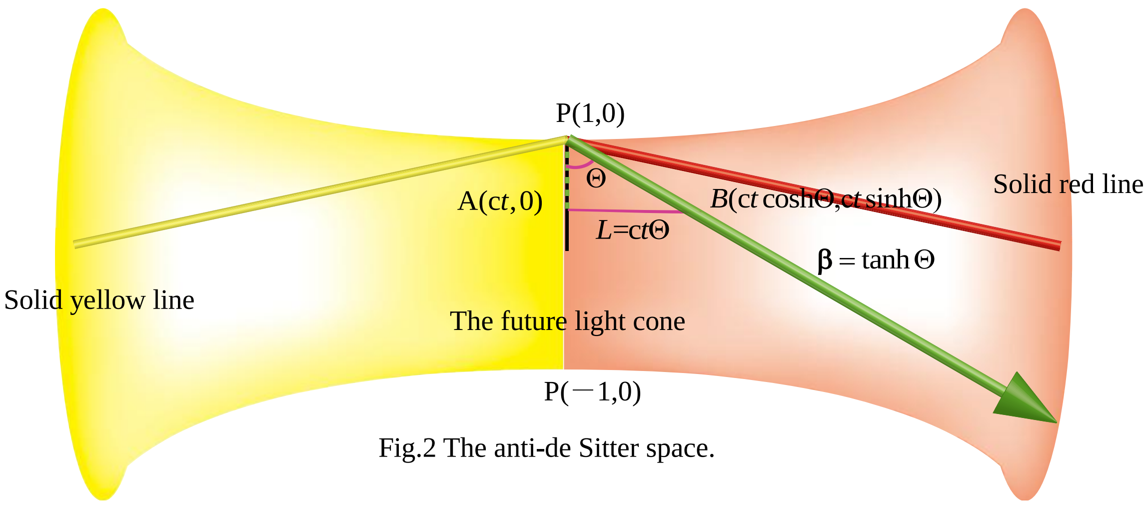

Fig.2 The anti-de Sitter space.

We keep this circumstances and roll up this Minkowski space such as closing the point $t=+\infty$, $t=-\infty$ to each other. The conservation of their simultaneous reach surfaces and null lines on Minkowski plane accords $t=\pm\infty$. It is conceived of the image of an anti-de Sitter space. Figure 2 shows a schematic diagram of the anti-de Sitter space.

Hence, we get the anti-de Sitter space of which tangent space is the Minkowski space.

1.3 Maxwell's Equation and wave equation

We introduce the time−component E_t in the electromagnetic field $\boldsymbol{E}-i\mathrm{c}\boldsymbol{B}$, then we get the four−dimensional electromagnetic field for the derivative of the scalar potential ϕ and the vector potential $\mathrm{c}\boldsymbol{A}$ as follows:

$$
{}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-i\mathrm{c}\boldsymbol{B} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial \mathrm{c}t} & \\ & -\dfrac{\partial}{\partial \boldsymbol{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\boldsymbol{A} \end{bmatrix}^{+}
$$

$$
= {}^{-}\begin{bmatrix} \dfrac{\partial \phi}{\partial \mathrm{c}t} + \mathrm{div}\,\mathrm{c}\boldsymbol{A} & \\ & -\dfrac{\partial \mathrm{c}\boldsymbol{A}}{\partial \mathrm{c}t} - \mathrm{grad}\,\phi - i\,\mathrm{rot}\,\mathrm{c}\boldsymbol{A} \end{bmatrix}^{+}
$$

where $\partial/\partial\boldsymbol{r} = \partial/\partial x\,\mathbf{e}_1 + \partial/\partial y\,\mathbf{e}_2 + \partial/\partial z\,\mathbf{e}_3$ and $\partial/\partial \mathrm{c}t$ are differential operators. $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are orthogonal basis vectors.

$$
{}^{+}\begin{bmatrix} \dfrac{\partial}{\partial \mathrm{c}t} & \\ & \dfrac{\partial}{\partial \boldsymbol{r}} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-i\mathrm{c}\boldsymbol{B} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \partial \mathrm{c}t^2 - \partial \boldsymbol{r}^2 & \\ & 0 \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\boldsymbol{A} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \dfrac{\rho}{\varepsilon} & \\ & -\dfrac{\boldsymbol{j}_s}{\varepsilon \mathrm{c}} \end{bmatrix}^{+} .
$$

Moreover, we call a scalar density $\rho = \rho_0 u_0/\mathrm{c}$ and a vector density $\boldsymbol{j}_s/\mathrm{c} = \rho_0\boldsymbol{u}/\mathrm{c}$ as charge density and current density stream. ε is an electric permittivity of the medium. These components all together of four−dimensional vector compose a physical quantity.

Therefore, when $\rho = 0$, $\boldsymbol{j}_s = 0$, then we get the wave equation.

$$
{}^{+}\begin{bmatrix} \partial \mathrm{c}t^2-\partial \boldsymbol{r}^2 & \\ & 0\end{bmatrix}^{-}\,{}^{+}\begin{bmatrix}\phi & \\ & -\mathrm{c}\boldsymbol{A}\end{bmatrix}^{+} = {}^{+}\begin{bmatrix}0 & \\ & 0\end{bmatrix}^{+}.
$$

Thus

$$
{}^{-}\begin{bmatrix} \partial \mathrm{c}t^2-\partial \boldsymbol{r}^2 & \\ & 0\end{bmatrix}^{+}\,{}^{-}\begin{bmatrix}E_t & \\ & \boldsymbol{E}-\mathrm{i}\mathrm{c}\boldsymbol{B}\end{bmatrix}^{+} = {}^{-}\begin{bmatrix}\dfrac{\partial}{\partial \mathrm{c}t} & \\ & -\dfrac{\partial}{\partial \boldsymbol{r}}\end{bmatrix}^{-}\,{}^{+}\begin{bmatrix}\dfrac{\partial}{\partial \mathrm{c}t} & \\ & \dfrac{\partial}{\partial \boldsymbol{r}}\end{bmatrix}^{+}\,{}^{-}\begin{bmatrix}E_t & \\ & \boldsymbol{E}-\mathrm{i}\mathrm{c}\boldsymbol{B}\end{bmatrix}^{+} = {}^{-}\begin{bmatrix}0 & \\ & 0\end{bmatrix}^{+}.
$$

1.4 The four−dimensional Coulomb−Lorentz force

The four−dimensional electromagnetic field receives the four−dimensional force on the charge density ρ and the current density stream $\boldsymbol{j}_s$ as follows[1],[2]:

$$
{}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f}\end{bmatrix}^{-} = {}^{-}\begin{bmatrix}E_t & \\ & \boldsymbol{E}-\mathrm{i}\mathrm{c}\boldsymbol{B}\end{bmatrix}^{+}\,{}^{-}\begin{bmatrix}\rho & \\ & \dfrac{\boldsymbol{j}_s}{\mathrm{c}}\end{bmatrix}^{-}
$$

$$
= {}^{-}\begin{bmatrix} E_t\rho+\left(\boldsymbol{E}-\mathrm{i}\mathrm{c}\boldsymbol{B}\right)\bullet\dfrac{\boldsymbol{j}_s}{\mathrm{c}} & \\ & E_t\dfrac{\boldsymbol{j}_s}{\mathrm{c}}+\left(\boldsymbol{E}-\mathrm{i}\mathrm{c}\boldsymbol{B}\right)\cdot\rho-i\left(\boldsymbol{E}-\mathrm{i}\mathrm{c}\boldsymbol{B}\right)\times\dfrac{\boldsymbol{j}_s}{\mathrm{c}}\end{bmatrix}^{-}.
$$

2. The tension in the electromagnetic field

We have the formula as follows[1]:

$$
{}^{-}\begin{bmatrix}\dfrac{\partial}{\partial ct} & \\ & -\dfrac{\partial}{\partial r}\end{bmatrix}^{-}\,{}^{+}\begin{bmatrix}E_t & \\ & -\left(\boldsymbol{E}+\mathrm{i}\mathrm{c}\boldsymbol{B}\right)\end{bmatrix}^{-} = {}^{-}\begin{bmatrix}\dfrac{\rho}{\varepsilon} & \\ & \dfrac{\boldsymbol{j}_s}{\varepsilon \mathrm{c}}\end{bmatrix}^{-} \qquad (3)
$$

in the electromagnetic field ${}^{+}\begin{bmatrix}E_t & \\ & -\left(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B}\right)\end{bmatrix}^{-}$, then the four−dimensional force is

$$
{}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f}\end{bmatrix}^{-} = {}^{-}\begin{bmatrix}E_t & \\ & \boldsymbol{E}-\mathrm{i}\mathrm{c}\boldsymbol{B}\end{bmatrix}^{+}\,{}^{-}\begin{bmatrix}\rho & \\ & \dfrac{\boldsymbol{j}_s}{\mathrm{c}}\end{bmatrix}^{-}
$$

where we apply expression (3), and we have

$$
= {}^{-}\begin{bmatrix}\boldsymbol{E}_t & \\ & \boldsymbol{E}-i\mathrm{c}\boldsymbol{B}\end{bmatrix}^{+}\varepsilon\,{}^{-}\begin{bmatrix}\dfrac{\partial}{\partial \boldsymbol{ct}} & \\ & -\dfrac{\partial}{\partial r}\end{bmatrix}^{-}\,{}^{+}\begin{bmatrix}\boldsymbol{E}_t & \\ & -\left(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B}\right)\end{bmatrix}^{-}
$$

$$= \varepsilon \, {}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E} - \mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+} \begin{bmatrix} \dfrac{\partial E_t}{\partial ct} + \operatorname{div}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) & \\ & -\dfrac{\partial\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right)}{\partial ct} - \operatorname{grad} E_t - i \operatorname{rot}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) \end{bmatrix}^{-}$$

$$= \varepsilon \, {}^{-}\begin{bmatrix} \begin{gathered} E_t \left\{ \dfrac{\partial E_t}{\partial ct} + \operatorname{div}(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}) \right\} \\ -(\boldsymbol{E} - \mathrm{i}c\boldsymbol{B}) \bullet \left\{ \dfrac{\partial\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right)}{\partial ct} + \operatorname{grad} E_t + i \operatorname{rot}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) \right\} \end{gathered} & \\ & \begin{gathered} \left(\boldsymbol{E} - \mathrm{i}c\boldsymbol{B}\right) \cdot \left\{ \dfrac{\partial E_t}{\partial ct} + \operatorname{div}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) \right\} \\ -E_t \cdot \left\{ \dfrac{\partial\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right)}{\partial ct} + \operatorname{grad} E_t + i \operatorname{rot}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) \right\} \\ +i\left(\boldsymbol{E} - \mathrm{i}c\boldsymbol{B}\right) \times \left\{ \dfrac{\partial\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right)}{\partial ct} + \operatorname{grad} E_t + i \operatorname{rot}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) \right\} \end{gathered} \end{bmatrix}^{-} .$$

This means that

(i) The time component of the force f_t (the variation of energy) is

$$\begin{aligned} f_t &= E_t \rho + i\left(\boldsymbol{E} - \mathrm{i}c\boldsymbol{B}\right) \bullet \left(\boldsymbol{j}_s / \mathrm{c}\right) \\ &= \varepsilon \left(E_t \left\{ \frac{\partial E_t}{\partial ct} + \operatorname{div}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) \right\} - (\boldsymbol{E} - \mathrm{i}c\boldsymbol{B}) \bullet \left\{ \frac{\partial\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right)}{\partial ct} + \operatorname{grad} E_t + i \operatorname{rot}\left(\boldsymbol{E} + \mathrm{i}c\boldsymbol{B}\right) \right\} \right). \end{aligned}$$

Moreover this real part is

$$\begin{aligned} &\varepsilon \left(E_t \left(\frac{\partial E_t}{\partial ct} + \operatorname{div} \boldsymbol{E} \right) - \boldsymbol{E} \bullet \left(\frac{\partial \boldsymbol{E}}{\partial ct} + \operatorname{grad} E_t - \operatorname{rot} c\boldsymbol{B} \right) - c\boldsymbol{B} \bullet \left(\frac{\partial c\boldsymbol{B}}{\partial ct} + \operatorname{rot} \boldsymbol{E} \right) \right) \\ &= \varepsilon \left(\frac{1}{2} \frac{\partial}{\partial ct} \left\{ E_t^2 - \boldsymbol{E}^2 - (c\boldsymbol{B})^2 \right\} + \left(E_t \operatorname{div} \boldsymbol{E} - \boldsymbol{E} \bullet \operatorname{grad} E_t \right) - \operatorname{div}\left(\underline{\boldsymbol{E} \times c\boldsymbol{B}} \right) \right) \end{aligned}$$

where $\boldsymbol{S} = \underline{\boldsymbol{E} \times c\boldsymbol{B}}$ is a Poynting vector. Therefore, when the field is (i) stationary and (ii) $E_t = 0$, then this formula coincides with the divergence of a Poynting vector.

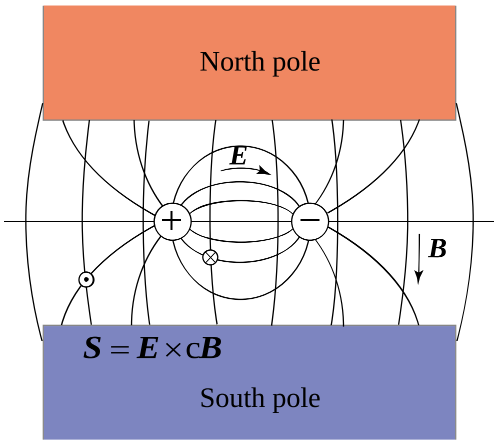

Fig. 3 Model geometry of the Poynting vector.

Example. This figure represents the Pointing vector $\boldsymbol{S}=\boldsymbol{E}\times \mathrm{c}\boldsymbol{B}\neq 0$

In this case, the field is stationary and Energy flow is free.

Therefore, $\operatorname{div}\boldsymbol{S}=E_t \operatorname{div}\boldsymbol{E}-\boldsymbol{E}\bullet\operatorname{grad}E_t$.

Because, in this stationary case, we use the formula

$$E_t=\frac{\partial\phi}{\partial \mathrm{c}t}+\operatorname{div}\mathrm{c}\boldsymbol{A}=\operatorname{div}\mathrm{c}\boldsymbol{A}\ ,\ \boldsymbol{E}-i\mathrm{c}\boldsymbol{B}=-\frac{\partial \mathrm{c}\boldsymbol{A}}{\partial \mathrm{c}t}-\operatorname{grad}\phi-i\operatorname{rot}\mathrm{c}\boldsymbol{A}=-\operatorname{grad}\phi-i\operatorname{rot}\mathrm{c}\boldsymbol{A}.$$

Figure 3 shows a schematic diagram of a model geometry of the Poynting vector.

Then

$$\begin{aligned}f_t&=-E_t\rho+(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B})\bullet(\boldsymbol{j}_s/\mathrm{c})\\&=\varepsilon\big(E_t\operatorname{div}(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B})-(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B})\bullet\{\operatorname{grad}E_t+i\operatorname{rot}(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B})\}\big)\\&=\varepsilon\big(E_t\operatorname{div}(-\operatorname{grad}\phi+i\operatorname{rot}\mathrm{c}\boldsymbol{A})-(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B})\bullet\{\operatorname{grad}\operatorname{div}\mathrm{c}\boldsymbol{A}+i\operatorname{rot}(-\operatorname{grad}\phi+i\operatorname{rot}\mathrm{c}\boldsymbol{A})\}\big)\\&=-\varepsilon E_t\square\phi-\varepsilon(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B})\bullet\square\boldsymbol{A}=0.\end{aligned}$$

(ii) The space component of the force (the variation of momentum) is

$$\begin{aligned}\boldsymbol{f}&=E_t\boldsymbol{j}_s/\mathrm{c}+\left(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B}\right)\cdot\rho-i\left(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B}\right)\times\boldsymbol{j}_s/\mathrm{c}\\&=\varepsilon\left(\left(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B}\right)\cdot\left\{\frac{\partial E_t}{\partial \mathrm{c}t}+\operatorname{div}(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B})\right\}-E_t\cdot\left\{\frac{\partial\left(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B}\right)}{\partial \mathrm{c}t}+\operatorname{grad}E_t+i\operatorname{rot}\left(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B}\right)\right\}\right.\\&\qquad\left.+i\left(\boldsymbol{E}-i\mathrm{c}\boldsymbol{B}\right)\times\left\{\frac{\partial\left(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B}\right)}{\partial \mathrm{c}t}+\operatorname{grad}E_t+i\operatorname{rot}\left(\boldsymbol{E}+i\mathrm{c}\boldsymbol{B}\right)\right\}\right).\end{aligned}$$

Furthermore this real part is $E_t\,\boldsymbol{j}_s/\mathrm{c}+\boldsymbol{E}\cdot\rho-\mathrm{c}\boldsymbol{B}\times\boldsymbol{j}_s/\mathrm{c}$

$$\begin{aligned}&=\varepsilon\left(\boldsymbol{E}\cdot\left(\frac{\partial E_t}{\partial \mathrm{c}t}+\operatorname{div}\boldsymbol{E}\right)+\mathrm{c}\boldsymbol{B}\cdot\operatorname{div}\mathrm{c}\boldsymbol{B}-E_t\cdot\left(\frac{\partial\boldsymbol{E}}{\partial \mathrm{c}t}+\operatorname{grad}E_t-\operatorname{rot}\mathrm{c}\boldsymbol{B}\right)\right.\\&\qquad\left.-\boldsymbol{E}\times\left(\frac{\partial \mathrm{c}\boldsymbol{B}}{\partial \mathrm{c}t}+\operatorname{rot}\boldsymbol{E}\right)+\mathrm{c}\boldsymbol{B}\times\left(\frac{\partial\boldsymbol{E}}{\partial \mathrm{c}t}+\operatorname{grad}E_t-\operatorname{rot}\mathrm{c}\boldsymbol{B}\right)\right)\\&=\varepsilon\left(-E_t\cdot\left(\frac{\partial\boldsymbol{E}}{\partial \mathrm{c}t}+\operatorname{grad}E_t-\operatorname{rot}\mathrm{c}\boldsymbol{B}\right)+\mathrm{c}\boldsymbol{B}\times\operatorname{grad}E_t+\boldsymbol{E}\cdot\frac{\partial E_t}{\partial \mathrm{c}t}-\boldsymbol{E}\times\frac{\partial \mathrm{c}\boldsymbol{B}}{\partial \mathrm{c}t}+\mathrm{c}\boldsymbol{B}\times\frac{\partial\boldsymbol{E}}{\partial \mathrm{c}t}\right.\\&\qquad\left.+\left(\underline{\boldsymbol{E}\cdot\operatorname{div}\boldsymbol{E}-\boldsymbol{E}\times\operatorname{rot}\boldsymbol{E}}\right)+\left(\underline{\underline{\mathrm{c}\boldsymbol{B}\cdot\operatorname{div}\mathrm{c}\boldsymbol{B}-\mathrm{c}\boldsymbol{B}\times\operatorname{rot}\mathrm{c}\boldsymbol{B}}}\right)\right)\end{aligned}$$

where $\varepsilon\Big((\underline{\boldsymbol{E}\cdot\operatorname{div}\boldsymbol{E}-\boldsymbol{E}\times\operatorname{rot}\boldsymbol{E}})+(\underline{\underline{\mathrm{c}\boldsymbol{B}\cdot\operatorname{div}\mathrm{c}\boldsymbol{B}-\mathrm{c}\boldsymbol{B}\times\operatorname{rot}\mathrm{c}\boldsymbol{B}}})\Big)$ is Maxwell tension.

Hence, when the field is (i) stationary and (ii) $E_t=0$, i.e. this formula coincides with the Maxwell tension.

3. Electromagnetic gravitational force

For simplicity, the mass “M”is stationary. Then from this potential $U=G_M M/r$, we generate the gravitational force as the same way as Coulomb−Lorentz force as follows[3]:

The gravitational field is

$$ {}^{-}\begin{bmatrix} \partial t & \\ & -\partial \mathbf{r} \end{bmatrix}^{-} \frac{G}{\mathrm{c}^2} \, {}^{+}\begin{bmatrix} \dfrac{M}{r} & \\ & 0 \end{bmatrix}^{+} = \frac{G}{\mathrm{c}^2} \, {}^{-}\begin{bmatrix} 0 & \\ & -\dfrac{\partial}{\partial \mathbf{r}}\left(\dfrac{M}{r}\right) \end{bmatrix}^{+} . $$

Therefore we get the four-dimensional gravitational force

$$ {}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = \frac{G}{\mathrm{c}^2} \, {}^{-}\begin{bmatrix} 0 & \\ & -\dfrac{\partial}{\partial \mathbf{r}}\left(\dfrac{M}{r}\right) \end{bmatrix}^{+} \frac{m_0}{\mathrm{c}} \, {}^{-}\begin{bmatrix} u_t & \\ & \mathbf{u} \end{bmatrix}^{-} $$

$$ = \frac{G m_0}{\mathrm{c}^3} \, {}^{-}\begin{bmatrix} -\dfrac{\partial}{\partial \mathbf{r}}\left(\dfrac{M}{r}\right)\cdot \mathbf{u} & \\ & -\dfrac{\partial}{\partial \mathbf{r}}\left(\dfrac{M}{r}\right) u_t + i \dfrac{\partial}{\partial \mathbf{r}}\left(\dfrac{M}{r}\right) \times \mathbf{u} \end{bmatrix}^{-} . $$

We call this force electromagnetic gravitation. We set $M_G = GM/\mathrm{c}^2$ where G is a gravitational constant, then we get Newton type equations of motion as

$$ \begin{cases} \dfrac{\mathrm{d}^2\, \mathrm{c}t}{\mathrm{d}\tau^2} = -\dfrac{M_G}{r^2}\left(\dfrac{\mathbf{r}}{r}\cdot\dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}\tau}\right)\dfrac{\mathrm{d}\,\mathrm{c}t}{\mathrm{d}\tau} \cdots (E1) \\ \dfrac{\mathrm{d}^2\, \mathbf{r}}{\mathrm{d}\tau^2} = -\dfrac{M_G}{r^2}\dfrac{\mathbf{r}}{r}\left(\dfrac{\mathrm{d}\,\mathrm{c}t}{\mathrm{d}\tau}\right)^2 + i\dfrac{M_G}{r^2}\left(\dfrac{\mathbf{r}}{r}\times\dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}\tau}\right)\dfrac{\mathrm{d}\,\mathrm{c}t}{\mathrm{d}\tau} \cdots (E2, E3, E4) \end{cases} . $$

Moreover the metric is Minkowski metric $\mathrm{d}s^2 = -\mathrm{d}\,\mathrm{c}t^2 + \mathrm{d}\mathbf{r}^2 = -\mathrm{d}\,\mathrm{c}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$.

In this system of equations, there are all information. We rewrite these equations of motion by the spherical polar coordinate (r, θ, ϕ), that is,

$$ \begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} . $$

Then we have the metric $\mathrm{d}s^2 = -\mathrm{d}\,\mathrm{c}t^2 + \mathrm{d}r^2 + r^2(\sin^2\theta\,\mathrm{d}\phi^2 + \mathrm{d}\theta^2)$.

Furthermore the equations of motion are

$$ (E1)\dfrac{\mathrm{d}^2\,\mathrm{c}t}{\mathrm{d}\tau^2} = -\dfrac{M_G}{\underline{r^2}}\left(\dfrac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\dfrac{\mathrm{d}\,\mathrm{c}t}{\mathrm{d}\tau}\right) \cdots \text{(the direction of time)}, $$

$$ (E2)\dfrac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\dfrac{M_G}{\underline{r^2}}\left(\dfrac{\mathrm{d}\,\mathrm{c}t}{\mathrm{d}\tau}\right)^2 + \dfrac{1}{\underline{\underline{r}}}\left\{\left(r\dfrac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)^2 + \left(r\sin\theta\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2\right\} \cdots \text{(the direction of radius)}, $$

$$ (E3)\dfrac{\mathrm{d}}{\mathrm{d}\tau}\left(r\dfrac{\mathrm{d}\theta}{\mathrm{d}\tau}\right) \underset{(real)}{=} -\dfrac{1}{\underline{\underline{r}}}\left(\dfrac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(r\dfrac{\mathrm{d}\theta}{\mathrm{d}\tau}\right) + \left(\cos\theta\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)\left(r\sin\theta\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}\right) \cdots \text{(the direction of longitude)}, $$

$$ (E4)\dfrac{\mathrm{d}}{\mathrm{d}\tau}\left(r\sin\theta\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}\right) \underset{(real)}{=} -\dfrac{1}{\underline{\underline{r}}}\left(\dfrac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(r\sin\theta\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}\right) - \left(\cos\theta\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)\left(r\dfrac{\mathrm{d}\theta}{\mathrm{d}\tau}\right) \cdots \text{(the direction of latitude)}, $$

and the following main equation $(E5)$ which decides the orbit on the equator of the Sun in the anti-de Sitter space. The subscript (real), the meaning of symbol $\underset{(real)}{=}$ is equal to only the real part.

$$(E5)\left(\frac{1}{\sqrt{e^{-2\frac{M_G}{r}}}}\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\Phi}\right)^2=\left(-\frac{\mathrm{c}^2}{C^2}+\frac{C_0{}^2}{C^2}\frac{1}{e^{-2\frac{M_G}{r}}}-\frac{1}{r^2}\right)e^{2\frac{M_G}{r}},\quad d\Phi=\sqrt{(\cosh\Omega d\phi)^2-(d\Omega)^2}$$ [4].

Theorem.

The Einstein field equation is $R_{\mu\nu}-Rg_{\mu\nu}/2=8\pi GT_{\mu\nu}/\mathrm{c}^4$ and the Schwarzschild metric (solution) is

$$(G)\,\mathrm{d}s^2=-\mathrm{c}^2\left(1-\frac{2M_G}{r}\right)\mathrm{d}t^2+\left(1-\frac{2M_G}{r}\right)^{-1}\mathrm{d}r^2+r^2\left(\sin^2\theta\,\mathrm{d}\phi^2+\mathrm{d}\theta^2\right)$$

$$=-\left(\sqrt{1-\frac{2M_G}{r}}\,\mathrm{d}\mathrm{c}t\right)^2+\left(\frac{1}{\sqrt{1-\frac{2M_G}{r}}}\mathrm{d}r\right)^2+r^2\left(\sin^2\theta\,\mathrm{d}\phi^2+\mathrm{d}\theta^2\right).$$

We take $\mathrm{d}'\mathrm{c}t=\sqrt{1-2M_G/r}\,\mathrm{d}\mathrm{c}t$ to $d\mathrm{c}t$ and $\mathrm{d}'r=1/\sqrt{1-2M_G/r}\,\mathrm{d}r$ to $\mathrm{d}r$ as corresponding. Then this metric corresponds to a Minkowski metric.

$$(E)\,\mathrm{d}s^2=-\mathrm{d}\mathrm{c}t^2+\mathrm{d}r^2+r^2\left(\sin^2\theta\,\mathrm{d}\phi^2+\mathrm{d}\theta^2\right).$$

Therefore, (G) and (E) are two metrics of the same style.

By the same correspondence, the equations of motion $(E1)$-$(E4)$ and $(E5)$ are very similar to the following ones $(G1)'$-$(G4)'$, $(G5)'$ of the general theory of relativity.

$$(G1)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\mathrm{d}'\mathrm{c}t}{\mathrm{d}\tau}\right)=-\frac{M_G}{r^2\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}'\mathrm{c}t}{\mathrm{d}\tau}\right)\cdots\text{(the direction of time)},$$

$$(G2)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)=-\frac{M_G}{r^2\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}'\mathrm{c}t}{\mathrm{d}\tau}\right)^2+\frac{1}{r}\sqrt{1-\frac{2M_G}{r}}\left\{\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)^2+\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2\right\}\cdots\text{(the direction of radius)},$$

$$(G3)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)=-\frac{1}{r}\sqrt{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)+\left(\cos\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)\cdots\text{(the direction of longitude)},$$

$$(G4)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)=-\frac{1}{r}\sqrt{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)-\left(\cos\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)\cdots\text{(the direction of latitude)},$$

and the main equation is

$$(G5)'\left(\frac{1}{\sqrt{1-\frac{2M_G}{r}}}\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\phi}\right)^2=-\frac{\mathrm{c}^2}{C^2}+\frac{C_0{}^2}{C^2}\frac{1}{1-\frac{2M_G}{r}}-\frac{1}{r^2}.$$

(Proof)

From the general relativity theory, the equations of the motion and the main equation are

$$(G1)\frac{\mathrm{d}^2\,\mathrm{c}t}{\mathrm{d}\tau^2}=-\frac{\frac{2M_G}{r^2}}{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}\mathrm{c}t}{\mathrm{d}\tau}\right),$$

$$(G2)\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2}=-\frac{M_G}{r^2}\left(1-\frac{2M_G}{r}\right)\left(\frac{\mathrm{d}ct}{\mathrm{d}\tau}\right)^2+\frac{\frac{M_G}{r^2}}{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right),$$

$$(G3)\frac{\mathrm{d}^2\theta}{\mathrm{d}\tau^2}=-\frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}+\sin\theta\cos\theta\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2,$$

$$(G4)\frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2}=-\frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\phi}{\mathrm{d}\tau}-2\cot\theta\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\phi}{\mathrm{d}\tau},\text{and}$$

$$(G5)\left(\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\phi}\right)^2=\frac{C_0{}^2}{C^2}-\frac{\mathrm{c}^2}{C^2}\left(1-\frac{2M_G}{r}\right)-\frac{1}{r^2}\left(1-\frac{2M_G}{r}\right).$$

We apply the formula (G1), and obtain

$$(G1)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\mathrm{d}'ct}{\mathrm{d}\tau}\right)=\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\sqrt{1-\frac{2M_G}{r}}\frac{\mathrm{d}ct}{\mathrm{d}\tau}\right)=\frac{\frac{2M_G}{r^2}}{2\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}ct}{\mathrm{d}\tau}\right)+\sqrt{1-\frac{2M_G}{r}}\frac{\mathrm{d}^2ct}{\mathrm{d}\tau^2}$$

$$=\frac{\frac{M_G}{r^2}}{\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}ct}{\mathrm{d}\tau}\right)-\frac{\frac{2M_G}{r^2}}{\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}ct}{\mathrm{d}\tau}\right)=-\frac{M_G}{r^2\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}'ct}{\mathrm{d}\tau}\right).$$

According to the formula (G2), we get

$$(G2)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)=\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{1}{\sqrt{1-\frac{2M_G}{r}}}\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)=-\frac{\frac{2M_G}{r^2}}{2(1-\frac{2M_G}{r})\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2+\frac{1}{\sqrt{1-\frac{2M_G}{r}}}\frac{\mathrm{d}^2r}{\mathrm{d}\tau^2}$$

$$=-\frac{\frac{M_G}{r^2}}{(\sqrt{1-\frac{2M_G}{r}})^3}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2-\frac{M_G}{r^2}\sqrt{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}ct}{\mathrm{d}\tau}\right)^2+\frac{\frac{M_G}{r^2}}{(\sqrt{1-\frac{2M_G}{r}})^3}\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)^2+r\sqrt{1-\frac{2M_G}{r}}\left\{\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)^2+\left(\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2\right\}$$

$$=-\frac{M_G}{r^2\sqrt{1-\frac{2M_G}{r}}}\left(\frac{\mathrm{d}'ct}{\mathrm{d}\tau}\right)^2+\frac{1}{r}\sqrt{1-\frac{2M_G}{r}}\left\{\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)^2+\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2\right\}.$$

From the formula (G3), we find that

$$(G3)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)=\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)+r\frac{\mathrm{d}^2\theta}{\mathrm{d}\tau^2}=\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)-2\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}+r\sin\theta\cos\theta\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2$$

$$=-\frac{1}{r}\sqrt{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)+\left(\cos\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right).$$

From the formula (G4), we see that

$$(G4)'\frac{\mathrm{d}}{\mathrm{d}\tau}\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)=\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)+r\cos\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)+r\sin\theta\left(\frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2}\right)$$

$$=\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)+r\cos\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)-2\sin\theta\left(\frac{\mathrm{d}r}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)-2r\cos\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)$$

$$=-\frac{1}{r}\sqrt{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}'r}{\mathrm{d}\tau}\right)\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)-\left(\cos\theta\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)\left(r\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right).$$

Consequently, at last from the formula (G5), we know the main equation

$$(G5)'\left(\frac{1}{\sqrt{1-\frac{2M_G}{r}}}\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\phi}\right)^2=\frac{1}{1-\frac{2M_G}{r}}\left(\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\phi}\right)^2=-\frac{\mathrm{c}^2}{C^2}+\frac{C_0{}^2}{C^2}\underline{\underline{\frac{1}{1-\frac{2M_G}{r}}}}-\frac{1}{r^2} \qquad \text{(Q.E.D.)}$$

This means that from two different ways, we get the very close results [3],[4] .

The little difference between the formulas are covered by the imaginary part of (E3), (E4).

Conclusion

The authors generate the gravitational force by imitating the four-dimensional Coulomb−Lorentz force, and investigate the similarity and the difference between “the general theory of relativity” and “the Electromagnetic and Gravitational theory”. The following conclusions are drawn:

(1) The authors mention the image of calculation of the matrix-vectors.

(2) It is shown that the figure of the imaginary angle Θ in the Lorentz form.

(3) It is demonstrated that the four-dimensional vector product in the electromagnetic field.

(4) It is put to practical use of the four-dimensional Coulomb−Lorentz force.

(5) The authors mention that the formulas between the general theory of relativity and the electromagnetic theory are very similar.

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