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The Electric Charge and the Imaginary Charge*

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Abstract

In the previous paper, we transformed the system of unit to the relativistic form, and found that the forces of coulomb and ampere are the same. We can take the common coefficient in the two forces. The next aim is the magnetic field and its source of magnetic charge.

We provided the imaginary charge and found that it means the magnetic monopole. We explain the difference between the dipole of magnetic charge and the dipole of the loop current. We consider the electric charge Q as a complex charge $q+iq'$ and define its potential and field.

We expect that the knowledge for "matrix vector", "relativistic form" and the Maxwell Equation is well known[1].

1. Introduction

1.1 The magnetic charge and its System of Unit

We assume the existence of the magnetic charge. Then the Coulomb's law of the magnetic charge is $F = k\dfrac{m_1 m_2}{r^2}$ (Gauss).

This force can divides two formulas as follows:

The force in the magnetic field B is $F = k_{(2)} m_2 B$, and the magnetic field is $B = k_{(1)}\dfrac{m_1}{r^2}$.

1.2 The magnetic dipole moment and magnetic loop dipole moment by a current loop

(i) The magnetic dipole moment $\boldsymbol{m}$ which points from the magnetic south pole towards the magnetic north pole, has a magnitude $2ml$, where the m is the strength of each magnetic pole and the $2l$ is the distance between two magnetic poles. And then the potential $\Omega_m(\mathbf{r})$ and the field $H_m(\mathbf{r})$ are

$$\Omega_m(\mathbf{r}) = \frac{m}{4\pi\mu_0}\frac{\boldsymbol{m}\bullet\mathbf{r}}{r^3}, H_m(\mathbf{r}) = -\frac{1}{4\pi\mu_0}[\frac{\boldsymbol{m}}{r^3} - \frac{3(\boldsymbol{m}\bullet\mathbf{r})\mathbf{r}}{r^5}].$$

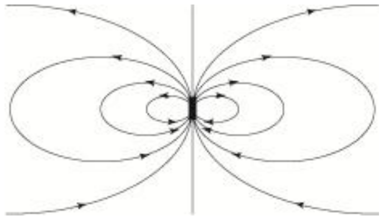


Fig. 1. The image of magnetic dipole.

(ii) For the current loop, the magnetic loop dipole moment is $\boldsymbol{m}_l = \mu_0 I\mathbf{S}$, where I is the current in the loop and $\mathbf{S}$ is an area of the loop. And then the magnetic field of the loop radius of which is a.

$$H_l(\mathbf{r}) = -\frac{Ia^2}{2\sqrt{(z^2+a^2)^3}}, r = \sqrt{z^2+a^2}.$$

Then $\boldsymbol{m} = \mu_0 I\pi a^2 = \boldsymbol{m}_l$.

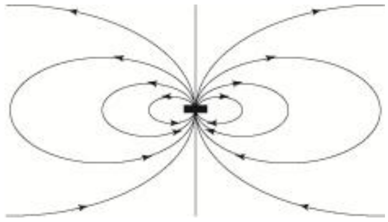


Fig. 2. The image of loop dipole.

2. The magnetic charge and the electric imaginary charge

2.1 The imaginary charge

We consider the electric charge Q as a complex charge $Q = q + \mathrm{i}q'$ formally, especially the pure imaginary charge $\mathrm{i}q'$. And we are going the same way as the real charge.

Then the pure imaginary charge $\mathrm{i}\overrightarrow{q'}$ and its potential as "en bloc" are

$$\mathrm{i}\overrightarrow{q'} = {}^{+}\begin{bmatrix} \mathrm{i}q_0'\gamma & \\ & -\mathrm{i}q_0'\gamma\boldsymbol{\beta} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \mathrm{i}q' & \\ & -\mathrm{i}\frac{\mathbf{I}'_s}{\mathrm{c}} \end{bmatrix}^{+} \text{ and}$$

$${}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\mathbf{A} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \mathrm{i}\frac{q_0'\gamma}{r} & \\ & -\mathrm{i}\frac{q_0'\gamma\boldsymbol{\beta}}{r} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \mathrm{i}\frac{q'}{r} & \\ & -\mathrm{i}\frac{\mathbf{I}'_s}{\mathrm{c}r} \end{bmatrix}^{+}.$$

Therefore, its electromagnetic field is $E_t = 0$ and

$$^{-}\begin{bmatrix} E_t(=0) - \mathrm{ic}B_t & \\ & \mathbf{E} - \mathrm{ic}\mathbf{B} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \frac{\partial}{\partial \mathrm{c}t} & \\ & -\frac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} \, {}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\mathbf{A} \end{bmatrix}^{+}$$

$$= {}^{-}\begin{bmatrix} \frac{\partial}{\partial \mathrm{c}t} & \\ & -\frac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} \, {}^{+}\begin{bmatrix} \mathrm{i}\frac{q'}{r} & \\ & -\mathrm{i}\frac{\mathbf{I}'_s}{\mathrm{c}r} \end{bmatrix}^{+}$$

$$= {}^{-}\begin{bmatrix} \mathrm{i}(\frac{\partial}{\partial \mathrm{c}t}\frac{q'}{r} + \mathbf{div}\frac{\mathbf{I}'_s}{\mathrm{c}r}) & \\ & \mathbf{rot}\frac{\mathbf{I}'_s}{\mathrm{c}r} - \mathrm{i}\mathbf{grad}\frac{q'}{r} \end{bmatrix}^{+}.$$

Therefore $B_t = \frac{\partial}{\partial \mathrm{c}t}\frac{q'}{r} + \mathbf{div}\frac{\mathbf{I}'_s}{\mathrm{c}r}$, $\mathbf{E} = \mathbf{rot}\frac{\mathbf{I}'_s}{\mathrm{c}r}$, $\mathrm{c}\mathbf{B} = \mathbf{grad}\frac{q'}{r}$,

And we can define the 4-dimensional force by using complex conjugate as

$$^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = \frac{1}{4\pi\varepsilon_0} \, {}^{-}\begin{bmatrix} -\mathrm{ic}B_t & \\ & \mathbf{E} - \mathrm{ic}\mathbf{B} \end{bmatrix}^{+} \, \overline{{}^{-}\begin{bmatrix} \mathrm{i}q'_2 & \\ & \frac{\mathrm{i}\mathbf{I}'_{s2}}{\mathrm{c}} \end{bmatrix}^{-}}$$

$$= \frac{1}{4\pi\varepsilon_0} \, {}^{-}\begin{bmatrix} -\mathrm{ic}B_t & \\ & \mathbf{E} - \mathrm{ic}\mathbf{B} \end{bmatrix}^{+} \, {}^{-}\begin{bmatrix} -\mathrm{i}q'_2 & \\ & -\frac{\mathrm{i}\mathbf{I}'_{s2}}{\mathrm{c}} \end{bmatrix}^{-},$$ $\frac{1}{4\pi\varepsilon_0}$ is the same as charge.

For simplicity, we assume that the pure imaginary charge is not moving, i.e. stationary. Then the charge, potential, electromagnetic field and force as "en bloc" are

$$\mathrm{i}\overrightarrow{q'_1} = {}^{+}\begin{bmatrix} \mathrm{i}q'_1 & \\ & \mathbf{0} \end{bmatrix}^{+}, \; {}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\mathbf{A} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \mathrm{i}\frac{q'_1}{r} & \\ & \mathbf{0} \end{bmatrix}^{+},$$

$$
{}^{-}\begin{bmatrix} -\mathrm{ic}B_t \\ \mathbf{E}-\mathrm{ic}\mathbf{B} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} 0 \\ -\mathrm{i}\,\mathbf{grad}\dfrac{q'_1}{r} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} 0 \\ \mathrm{i}\dfrac{q'_1\mathbf{r}}{r^3} \end{bmatrix}.
$$

The time component and the electric field are zero.

Moreover, $${}^{-}\begin{bmatrix} f_t \\ \boldsymbol{f} \end{bmatrix}^{-} = \frac{1}{4\pi\varepsilon_0}\, {}^{-}\begin{bmatrix} 0 \\ \mathrm{i}\dfrac{q'_1}{r^2}\dfrac{\mathbf{r}}{r} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} -\mathrm{i}q'_2 \\ \mathbf{0} \end{bmatrix}^{-} = \frac{1}{4\pi\varepsilon_0}\, {}^{-}\begin{bmatrix} 0 \\ \dfrac{q'_1 q'_2}{r^2}\dfrac{\mathbf{r}}{r} \end{bmatrix}^{-}.$$

Then the force, $f_t = 0$, $\boldsymbol{f} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q'_1 q'_2\mathbf{r}}{r^3}$, $\varepsilon_0\mu_0 = \dfrac{1}{\mathrm{c}^2}$

(1)

is a coulomb type force.

2.2 The identity with the magnetic charge

There exists the magnetic charge q_m, and the magnetic force between two magnetic charges q_{m1}, q_{m2} is

$\boldsymbol{F} = \dfrac{q_{m1}q_{m2}}{4\pi\mu_0}\dfrac{\boldsymbol{r}}{r^3} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_{m2}}{\mathrm{c}\mu_0}\underline{\dfrac{q_{m1}}{\mathrm{c}\mu_0}\dfrac{\boldsymbol{r}}{r^3}}$, this is the same type as the formula (1).

Moreover, we define the magnetic field $\boldsymbol{H}(\boldsymbol{r})$ as

$$
-\mathrm{ic}\boldsymbol{H}(\boldsymbol{r}) = -\mathrm{i}\underline{\frac{q_{m1}}{\mathrm{c}\mu_0}\frac{\boldsymbol{r}}{r^3}} = -\mathbf{grad}\left(-\frac{\mathrm{i}q_{m1}}{\mathrm{c}\mu_0}\frac{1}{r}\right) = -\mathbf{grad}\,\phi(\boldsymbol{r}),\quad \phi(\boldsymbol{r}) = -\frac{\mathrm{i}q_{m1}}{\mathrm{c}\mu_0 r}.
$$

Then,

$$
\boldsymbol{F} = \mathrm{i}\frac{\mu_0}{4\pi}\frac{q_{m2}}{\mathrm{c}\mu_0}\cdot\left[-\mathrm{i}\underline{\frac{q_{m1}}{\mathrm{c}\mu_0}\frac{\boldsymbol{r}}{r^3}}\right] = \mathrm{i}\frac{\mu_0}{4\pi}\frac{q_{m2}}{\mathrm{c}\mu_0}\cdot[-\mathrm{ic}\boldsymbol{H}(\boldsymbol{r})].
$$

This is the same type as pure imaginary charge, and then it is also the potential and the magnetic field. Therefore, we identify the imaginary part of the charge q' with the magnetic charge $-\dfrac{q_m}{\mathrm{c}\mu_0}$.

And then $\phi = \mathrm{i}\dfrac{q'}{r}$ and $\phi(\boldsymbol{r}) = -\dfrac{\mathrm{i}q_m}{\mathrm{c}\mu_0 r}$, $-\mathrm{ic}\mathbf{B} = \mathrm{i}\dfrac{q'}{r^2}\dfrac{\mathbf{r}}{r}$ and $-\mathrm{i}c\boldsymbol{H}(\boldsymbol{r}) = -\dfrac{\mathrm{i}q_m}{\mathrm{c}\mu_0 r^2}\dfrac{\boldsymbol{r}}{r}$ correspond to each other. That is to say, this is a representation of the magnetic monopole (or magnetic charge) as mathematics.

3. The magnetic dipole moment and the magnetic loop dipole moment

3.1 The magnetic dipole moment

We consider the magnetic charge m apart from a center with distance l and we put this vector $\boldsymbol{l} = (l_x, l_y, l_z)$, $l = |\boldsymbol{l}| = \sqrt{(l_x)^2 + (l_y)^2 + (l_z)^2}$. And we put the other two vectors from the magnetic charge or center to any point P which are $\mathbf{r}(P)$ and $\mathbf{R}(P)$ respectively. And then $\boldsymbol{l} = \mathbf{R}(P) - \mathbf{r}(P)$.

Next we calculate the magnetic dipole moment of two magnetic charges $\mathrm{i}q$ and $-\mathrm{i}q$ with distance $2l$ as follows:

We fixed the point P and move the point of magnetic charge $\boldsymbol{l}$.

The absolute value of $\mathbf{r}(\boldsymbol{l}) = \mathbf{R} - \boldsymbol{l}$ is

$$r = |\mathbf{r}| = \sqrt{(R_x - l_x)^2 + (R_y - l_y)^2 + (R_z - l_z)^2}$$

$$= \sqrt{R_x{}^2 + R_y{}^2 + R_z{}^2} + \frac{-2R_x l_x - 2R_y l_y - 2R_z l_z}{2\sqrt{R_x{}^2 + R_y{}^2 + R_z{}^2}} + \cdots$$

$$= R - \frac{R_x l_x + R_y l_y + R_z l_z}{R} + \cdots, \quad R = |\mathbf{R}| = \sqrt{R_x{}^2 + R_y{}^2 + R_z{}^2}$$

$$= R\Big(1 - \underline{\underline{\frac{R_x l_x + R_y l_y + R_z l_z}{R^2}}} + \cdots\Big).$$

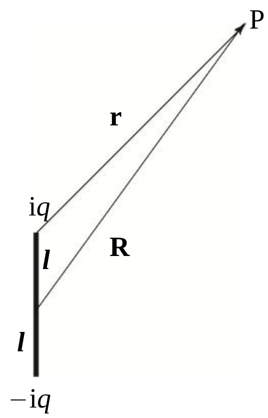


Fig. 3. The image of magnetic dipole.

Generally, we use the "en bloc" formulas.

$$
{}^{-}\begin{bmatrix} ct & \\ & \mathbf{r} \end{bmatrix}^{-}\,{}^{+}\begin{bmatrix} ct & \\ & -\mathbf{r} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} (ct)^2-\mathbf{r}^2 & \\ & \mathbf{0} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} (ct)^2-(x^2+y^2+z^2) & \\ & \mathbf{0} \end{bmatrix}^{+}, ct=0 \,.
$$

Then the absolute value $|\mathbf{r}|$ is represented as ${}^{-}\begin{bmatrix} |\mathbf{r}| & \\ & \mathbf{0} \end{bmatrix}^{+}$ by “en bloc”.

We use the (relativistic) total differential, and then the double underlined part above is approximate to the following formula.

$$
\mathrm{d}\;{}^{-}\begin{bmatrix} |\mathbf{r}(\boldsymbol{l})| & \\ & \mathbf{0} \end{bmatrix}^{+} = \mathrm{d}\;{}^{-}\begin{bmatrix} |\mathbf{R}-\boldsymbol{l}| & \\ & \mathbf{0} \end{bmatrix}^{+}, \quad \mathrm{d}\mathbf{r} = \mathrm{d}(\mathbf{R}-\boldsymbol{l}) = -\mathrm{d}\boldsymbol{l}
$$

$$
= {}^{-}\begin{bmatrix} \mathrm{d}ct & \\ & \mathrm{d}\mathbf{r} \end{bmatrix}^{-} \left({}^{+}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & \dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} |\mathbf{R}-\boldsymbol{l}| & \\ & \mathbf{0} \end{bmatrix}^{+} \right) = -\;{}^{-}\begin{bmatrix} \mathrm{d}ct(=0) & \\ & \mathrm{d}\mathbf{r} \end{bmatrix}^{-}\;{}^{+}\begin{bmatrix} 0 & \\ & -\dfrac{\mathbf{R}-\boldsymbol{l}}{|\mathbf{R}-\boldsymbol{l}|} \end{bmatrix}^{+} .
$$

In this situation, we replace $\mathbf{s}$ as $\mathbf{s} = \mathrm{d}\boldsymbol{l} = -\mathrm{d}\mathbf{r}$ and $\boldsymbol{l} = \mathbf{0}$.

$$
\mathrm{d}\;{}^{-}\begin{bmatrix} |\mathbf{r}| & \\ & 0 \end{bmatrix}^{+} = -\;{}^{-}\begin{bmatrix} 0 & \\ & \mathrm{d}\mathbf{r} \end{bmatrix}^{-}\;{}^{+}\begin{bmatrix} 0 & \\ & -\dfrac{\mathbf{R}}{|\mathbf{R}|} \end{bmatrix}^{+}
$$

$$
= {}^{-}\begin{bmatrix} \mathrm{d}\mathbf{r}\bullet\dfrac{\mathbf{R}}{|\mathbf{R}|} & \\ & -\mathrm{i}\left(\mathrm{d}\mathbf{r}\times\dfrac{\mathbf{R}}{|\mathbf{R}|}\right) \end{bmatrix}^{+}
$$

$$
= -\;{}^{-}\begin{bmatrix} \underline{\underline{\dfrac{\mathbf{s}\bullet\mathbf{R}}{|\mathbf{R}|}}} & \\ & -\mathrm{i}\,\dfrac{\mathbf{s}\times\mathbf{R}}{|\mathbf{R}|} \end{bmatrix}^{+} .
$$

By the way, the value of potential is proportional to $\frac{1}{r}$, and its variation by $\boldsymbol{l}$ is

$$\frac{1}{r}=\underline{\underline{\frac{1}{R}}}(1+\underline{\underline{\frac{R_x l_x+R_y l_y+R_z l_z}{R^2}}}+\bullet\bullet\bullet)=\underline{\underline{\frac{1}{R}}}(1+\underline{\underline{\frac{\mathbf{R}\bullet\boldsymbol{l}}{R^2}}}+\bullet\bullet\bullet).$$

And this underlined part is an approximate time component of the following “en bloc” formula (3).

$$\mathrm{d}\ {}^{-}\begin{bmatrix}\frac{1}{|\mathbf{r}|} & \\ & \mathbf{0}\end{bmatrix}^{+}=\mathrm{d}\ {}^{-}\begin{bmatrix}\frac{1}{|\mathbf{R}-\boldsymbol{l}|} & \\ & \mathbf{0}\end{bmatrix}^{+}={}^{-}\begin{bmatrix}\mathrm{d}\mathrm{c}t(=0) & \\ & \mathrm{d}\mathbf{r}\end{bmatrix}^{-}\left({}^{+}\begin{bmatrix}\frac{\partial}{\partial \mathrm{c}t} & \\ & \frac{\partial}{\partial \mathbf{r}}\end{bmatrix}^{+}\ {}^{-}\begin{bmatrix}\frac{1}{|\mathbf{R}-\boldsymbol{l}|} & \\ & \mathbf{0}\end{bmatrix}^{+}\right)$$

$$={}^{-}\begin{bmatrix}\mathrm{d}\mathrm{c}t & \\ & \mathrm{d}\mathbf{r}\end{bmatrix}^{-}\ {}^{+}\begin{bmatrix}0 & \\ & -\frac{\mathbf{R}-\boldsymbol{l}}{|\mathbf{R}-\boldsymbol{l}|^3}\end{bmatrix}^{+}. \tag{2}$$

In this situation, we replace $\mathbf{s}$ as $\mathbf{s}=\mathrm{d}\boldsymbol{l}=-\mathrm{d}\mathbf{r}$ and $\boldsymbol{l}=\mathbf{0}$.

$$\mathrm{d}\ {}^{-}\begin{bmatrix}\frac{1}{|\mathbf{r}|} & \\ & 0\end{bmatrix}^{+}=-\ {}^{-}\begin{bmatrix}0 & \\ & \mathbf{s}\end{bmatrix}^{-}\ {}^{+}\begin{bmatrix}0 & \\ & -\frac{\mathbf{R}}{|\mathbf{R}|^3}\end{bmatrix}^{+}={}^{-}\begin{bmatrix}\underline{\underline{\frac{\mathbf{s}\bullet\mathbf{R}}{|\mathbf{R}|^3}}} & \\ & -\mathrm{i}\frac{\mathbf{s}\times\mathbf{R}}{|\mathbf{R}|^3}\end{bmatrix}^{+}. \tag{3}$$

Therefore, by use of formula (3), and we subtract the potential of magnetic charge $\mathrm{i}q'(=-\frac{\mathrm{i}q_m}{\mathrm{c}\mu_0})$ from the potential of $-\mathrm{i}q'$. Then the potential of magnetic dipole is

$${}^{+}\begin{bmatrix}\phi & \\ & -\mathrm{c}\mathbf{A}\end{bmatrix}^{+}=2\ {}^{+}\begin{bmatrix}-\frac{\mathrm{i}q_m}{\mathrm{c}\mu_0} & \\ & \mathbf{0}\end{bmatrix}^{+}\ {}^{-}\begin{bmatrix}\frac{\mathbf{s}\bullet\mathbf{R}}{|\mathbf{R}|^3} & \\ & -\mathrm{i}\frac{\mathbf{s}\times\mathbf{R}}{|\mathbf{R}|^3}\end{bmatrix}^{+}$$

$$= -2 \, {}^{+}\begin{bmatrix} -\dfrac{\mathrm{i}q_m}{\mathrm{c}\mu_0} & \\ & \mathbf{0} \end{bmatrix}^{+} \, {}^{-}\begin{bmatrix} 0 & \\ & \mathbf{s} \end{bmatrix}^{-} \, {}^{+}\begin{bmatrix} 0 & \\ & -\dfrac{\mathbf{R}}{|\mathbf{R}|^3} \end{bmatrix}^{+}$$

$$= - \, {}^{+}\begin{bmatrix} 0 & \\ & -2\dfrac{\mathrm{i}q_m}{\mathrm{c}\mu_0}\mathbf{s} \end{bmatrix}^{-} \, {}^{+}\begin{bmatrix} 0 & \\ & -\dfrac{\mathbf{R}}{|\mathbf{R}|^3} \end{bmatrix}^{+}$$

$$= - \, {}^{+}\begin{bmatrix} -\mathrm{i}\dfrac{\mathbf{m} \bullet \mathbf{R}}{|\mathbf{R}|^3} & \\ & -\dfrac{\mathbf{m} \times \mathbf{R}}{|\mathbf{R}|^3} \end{bmatrix}^{+}, \mathbf{m} = -2\frac{q_m}{\mathrm{c}\mu_0}\mathbf{s} \,. \tag{4}$$

Where, m is magnetic charge, $\mathbf{m}$ is magnetic dipole vector, $2\mathbf{s}$ is the distance between magnetic charge $\mathrm{i}q'(=-\frac{\mathrm{i}q_m}{\mathrm{c}\mu_0})$ and $-\mathrm{i}q'$.

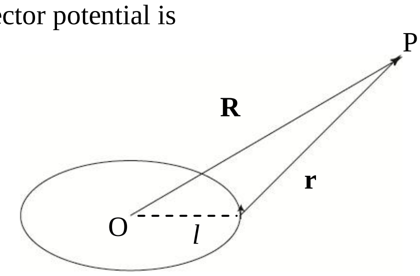
3.2 The magnetic loop dipole moment

We consider the current I (C/s) in conductor and moving charges with speed $\boldsymbol{v}$ (m/s). Then the current is $I = \rho_q \boldsymbol{v}\mathrm{d}\mathbf{s}$ (C/s) .

For simplicity, we assume that the charge $q = \rho_q \mathrm{d}x\mathrm{d}\boldsymbol{s}$ and its speed are homogeneous[3].

We take the current $\mathbf{I}$ in a loop with radius $\boldsymbol{l}$, then its vector potential is

$$\mathrm{c}\mathbf{A} = \oint \frac{|\mathbf{I}|}{\mathrm{c}r}\mathrm{d}\mathbf{s} = \oint \frac{|\mathbf{I}|}{\mathrm{c}R}\left(1 + \frac{\mathbf{R} \bullet \boldsymbol{l}}{R^2} + \bullet\bullet\bullet\right)\mathrm{d}\mathbf{s} \,,$$



$$\sim \frac{|\mathbf{I}|}{\mathrm{c}R^3}\oint(\mathbf{R} \bullet \boldsymbol{l})\mathrm{d}\mathbf{s} = \frac{|\mathbf{I}|}{\mathrm{c}R^3}\frac{1}{2}\oint\{(\mathbf{R} \bullet \boldsymbol{l})\mathrm{d}\mathbf{s} - (\mathbf{R} \bullet \mathrm{d}\mathbf{s})\boldsymbol{l}\}$$

Fig. 4. The image of loop dipole.

$$= \frac{|\mathbf{I}|}{\mathrm{c}R^3}\frac{1}{2}\oint(\boldsymbol{l}\times\mathrm{d}\mathbf{s})\times\mathbf{R} = \underline{\underline{\frac{\mathbf{m}\times\mathbf{R}}{R^3}}}, \quad \mathbf{m} = \frac{|\mathbf{I}|}{2\mathrm{c}}\oint(\boldsymbol{l}\times\mathrm{d}\mathbf{s}) \quad .$$

And this underlined part is an approximate space component of the following "en bloc" formula (5).

$${}^{+}\begin{bmatrix}\phi & \\ & -\mathrm{c}\mathbf{A}\end{bmatrix}^{+} = \oint {}^{+}\begin{bmatrix}0 & \\ & -\frac{|I|}{2\mathrm{c}}\mathrm{d}\mathbf{s}\end{bmatrix}^{+} \mathrm{d}\ {}^{-}\begin{bmatrix}\frac{1}{|\mathbf{r}|} & \\ & \mathbf{0}\end{bmatrix}^{+} = \oint {}^{+}\begin{bmatrix}0 & \\ & -\frac{|I|}{2\mathrm{c}}\mathrm{d}\mathbf{s}\end{bmatrix}^{+} \mathrm{d}\ {}^{-}\begin{bmatrix}\frac{1}{|\mathbf{R}-\boldsymbol{l}|} & \\ & \mathbf{0}\end{bmatrix}^{+}$$

$$= -\oint {}^{+}\begin{bmatrix}0 & \\ & \frac{|I|}{2\mathrm{c}}\mathrm{d}\mathbf{s}\end{bmatrix}^{+} {}^{-}\begin{bmatrix}\mathrm{d}\mathrm{c}t(=0) & \\ & \mathrm{d}\mathbf{r}\end{bmatrix}^{-} {}^{+}\begin{bmatrix}0 & \\ & -\frac{\mathbf{R}-\boldsymbol{l}}{|\mathbf{R}-\boldsymbol{l}|^3}\end{bmatrix}^{+} \quad . \qquad (5)$$

We take the same way as section 3.1. We replace $\mathbf{s}$ as $\mathbf{s} = \mathrm{d}\boldsymbol{l} = -\mathrm{d}\mathbf{r}$ and $\boldsymbol{l} = \mathbf{0}$. Therefore,

$$= -\oint {}^{+}\begin{bmatrix}0 & \\ & -\frac{|I|}{2\mathrm{c}}\mathrm{d}\mathbf{s}\end{bmatrix}^{+} {}^{-}\begin{bmatrix}0 & \\ & \mathbf{s}\end{bmatrix}^{-} {}^{+}\begin{bmatrix}0 & \\ & -\frac{\mathbf{R}}{|\mathbf{R}|^3}\end{bmatrix}^{+}, \oint \mathrm{d}s = 2\pi s \quad , s = |\mathbf{s}|$$

$$= {}^{+}\begin{bmatrix}0 & \\ & -\mathrm{i}\frac{|I|}{2\mathrm{c}}\oint\mathbf{s}\times\mathrm{d}\mathbf{s}\end{bmatrix}^{-} {}^{+}\begin{bmatrix}0 & \\ & -\frac{\mathbf{R}}{|\mathbf{R}|^3}\end{bmatrix}^{+}, \mathbf{m} = \frac{|I|}{2\mathrm{c}}\oint\mathbf{s}\times\mathrm{d}\mathbf{s}$$

$$= {}^{+}\begin{bmatrix}0 & \\ & -\mathrm{i}\mathbf{m}\end{bmatrix}^{-} {}^{+}\begin{bmatrix}0 & \\ & -\frac{\mathbf{R}}{|\mathbf{R}|^3}\end{bmatrix}^{+}$$

$$= -\,{}^{+}\begin{bmatrix} -\mathrm{i}\,\dfrac{\mathbf{m}\bullet\mathbf{R}}{|\mathbf{R}|^{3}} & \\ & \underline{\underline{-\dfrac{\mathbf{m}\times\mathbf{R}}{|\mathbf{R}|^{3}}}} \end{bmatrix}^{+} . \qquad (6)$$

Therefore, from (4) in 3.1 or (6).

$${}^{+}\begin{bmatrix} -\mathrm{i}cB_{t} & \\ & \mathbf{E}-ic\mathbf{B} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & \dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} \; {}^{+}\begin{bmatrix} \mathrm{i}\,\dfrac{\mathbf{m}\bullet\mathbf{R}}{|\mathbf{R}|^{3}} & \\ & \dfrac{\mathbf{m}\times\mathbf{R}}{|\mathbf{R}|^{3}} \end{bmatrix}^{+}$$

$$= {}^{-}\begin{bmatrix} 0 & \\ & \mathrm{i}\,\mathbf{grad}\dfrac{\mathbf{m}\bullet\mathbf{R}}{|\mathbf{R}|^{3}} - \mathrm{i}\,\mathbf{rot}\dfrac{\mathbf{m}\times\mathbf{R}}{|\mathbf{R}|^{3}} \end{bmatrix}^{+}$$

$$= 2\,{}^{-}\begin{bmatrix} 0 & \\ & \mathrm{i}[\mathbf{m}\bullet\mathbf{grad}]\dfrac{\mathbf{R}}{|\mathbf{R}|^{3}} \end{bmatrix}^{+} ,$$

because, we use $\dfrac{\partial \mathbf{m}}{\partial x} = \dfrac{\partial \mathbf{m}}{\partial y} = \dfrac{\partial \mathbf{m}}{\partial z} = \mathbf{0}$, $\mathbf{grad} = (\dfrac{\partial}{\partial x}, \dfrac{\partial}{\partial y}, \dfrac{\partial}{\partial z})$.

$$\mathbf{grad}\frac{\mathbf{m}\bullet\mathbf{R}}{|\mathbf{R}|^{3}} = [\mathbf{m}\bullet\mathbf{grad}]\frac{\mathbf{R}}{|\mathbf{R}|^{3}} + \mathbf{m}\times\mathbf{rot}(\mathbf{grad}\frac{1}{|\mathbf{R}|}) = [\mathbf{m}\bullet\mathbf{grad}]\frac{\mathbf{R}}{|\mathbf{R}|^{3}} ,$$

and $\mathbf{rot}\dfrac{\mathbf{m}\times\mathbf{R}}{|\mathbf{R}|^{3}} = -[\mathbf{m}\bullet\mathbf{grad}]\dfrac{\mathbf{R}}{|\mathbf{R}|^{3}} + \mathbf{m}\; div(\mathbf{grad}\dfrac{1}{|\mathbf{R}|}) = -[\mathbf{m}\bullet\mathbf{grad}]\dfrac{\mathbf{R}}{|\mathbf{R}|^{3}}$.

This means that the magnetic field of magnetic loop dipole has twice the value of

$$\mathrm{c}\mathbf{B} = [\mathbf{m}\bullet\mathbf{grad}]\frac{\mathbf{R}}{|\mathbf{R}|^3}.$$

cf. (Einstein-de Haas effect)

The magnetic loop dipole moment is $\mathbf{m} = \frac{|I|}{2}\oint(\boldsymbol{l}\times \mathrm{d}\mathbf{s}) = \frac{q\omega}{4\pi}\oint(\boldsymbol{l}\times \mathrm{d}\mathbf{s}), |I| = \frac{q\omega}{2\pi}$.

And the angular momentum is $\mathbf{I} = m_e \boldsymbol{l}\times\mathbf{v} = \frac{m_e\omega}{2\pi}\oint(\boldsymbol{l}\times \mathrm{d}\mathbf{s})$.

Therefore $\mathbf{m} = \frac{q}{2m_e}\mathbf{I}, \frac{q}{2m_e} = 8.79\times 10^{10}\, coulomb/kg$.

Then the ratio $\frac{q}{2m_e}$ of the magnetic loop dipole moment to the angular momentum remains unchanged.

3.3 The couples of magnetic dipole and magnetic loop dipole in the magnetic field

The couple of force of magnetic dipole is

$$^{-}\begin{bmatrix} N_0 & \\ & -\mathrm{i}\mathbf{N} \end{bmatrix}^{+} = 2\ {}^{-}\begin{bmatrix} 0 & \\ & \mathbf{F} \end{bmatrix}^{-}\ {}^{+}\begin{bmatrix} 0 & \\ & -\mathbf{s} \end{bmatrix}^{+}$$

$$= -\frac{1}{4\pi\varepsilon_0}\ {}^{-}\begin{bmatrix} 0 & \\ & -\mathrm{ic}\mathbf{B} \end{bmatrix}^{+}\cdot 2\ {}^{-}\begin{bmatrix} \mathrm{i}\frac{m}{\mathrm{c}} & \\ & \mathbf{0} \end{bmatrix}^{-}\ {}^{+}\begin{bmatrix} 0 & \\ & -\mathbf{s} \end{bmatrix}^{+}$$

$$= -\frac{1}{4\pi\varepsilon_0}\ {}^{-}\begin{bmatrix} 0 & \\ & -\mathrm{ic}\mathbf{B} \end{bmatrix}^{+}\ {}^{-}\begin{bmatrix} 0 & \\ & -\mathrm{i}\mathbf{m} \end{bmatrix}^{+}, \mathbf{m} = -2\frac{q_m}{\mathrm{c}\mu_0}\mathbf{s}$$

We get twice as much value in this formula.

The couple of force of magnetic loop dipole is

$$
{}^{-}\begin{bmatrix} N_0 & \\ & -\mathrm{i}\mathbf{N} \end{bmatrix}^{+} = \frac{1}{2}\oint {}^{-}\begin{bmatrix} 0 & \\ & \mathrm{d}\mathbf{F} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} 0 & \\ & -\mathbf{s} \end{bmatrix}^{+}
$$

$$
= -\frac{1}{4\pi\varepsilon_0}\oint {}^{-}\begin{bmatrix} 0 & \\ & -\mathrm{ic}\mathbf{B} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} 0 & \\ & \dfrac{I}{2\mathrm{c}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} 0 & \\ & -\mathbf{s} \end{bmatrix}^{+} \mathrm{d}s\,, |I|\mathrm{d}\mathbf{s} = I\mathrm{d}s
$$

$$
= -\frac{1}{4\pi\varepsilon_0} {}^{-}\begin{bmatrix} 0 & \\ & -\mathrm{ic}\mathbf{B} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} 0 & \\ & -\mathrm{i}\mathbf{m} \end{bmatrix}^{+}, \mathbf{m} = \frac{|I|}{2\mathrm{c}}\oint \mathbf{s}\times \mathrm{d}\mathbf{s} \, .
$$

We get twice as much value in this formula.

4. Conclusion

One point is that in this time, the magnetic monopole is not found. But we can identify the pure imaginary charge with the monopole. Then we find that it works as a same action which is expected as a monopole.

Another point is that in the quantum mechanism magnitude of spin moment is about twice the magnitude of the orbital angular momentum. But in this paper, we calculate the twice as much value of the orbital angular momentum. This means Lande g-factor is about 1 value.

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