~~電子の軌道とマクスウェル方程式~~

The Orbit of the Electron and the Maxwell Equation*

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Abstract

In our previous paper, we got the equation of motion from the Maxwell equation. In this paper, we proceed to calculate an electron orbit around the proton in the Hydrogen. We define the "**acceleration quantity**" and propose the mechanism by which the light is generated. In this mechanism, we can clarify the relation between the frequency of radiated light and the surrounding frequency energy of electron.

1. Introduction

We modify the Maxwell equation and find the equation of motion of electron in the Hydrogen. We take Bohr's image which looks like a planet in the solar system.

1.1 Preliminaries

The existence of the time component E_t in the electromagnetic field and the 4-dimensional electromagnetic field $(E_t, \boldsymbol{E} - \mathrm{i}c\boldsymbol{B})$ is obtained by derivation of the scalar potential ϕ and the vector potential $c\boldsymbol{A}$ as follows:

$$
{}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E} - \mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & -\dfrac{\partial}{\partial \boldsymbol{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \phi & \\ & -c\boldsymbol{A} \end{bmatrix}^{+}
$$

$$
= {}^{-}\begin{bmatrix} \dfrac{\partial \phi}{\partial ct} + div\, c\boldsymbol{A} & \\ & -\dfrac{\partial c\boldsymbol{A}}{\partial ct} - grad\,\phi - \mathrm{i}\,rot\, c\boldsymbol{A} \end{bmatrix}^{+} . \tag{1.1.1}
$$

The 4-dimensional force $(f_t, \boldsymbol{f})$ generated by the electromagnetic field $(E_t, \boldsymbol{E} - \mathrm{i}c\boldsymbol{B})$ on the moving charge $(u_t,\boldsymbol{u})$ is obtained as follows:

$$
{}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial \mathrm{c}t} & \\ & -\dfrac{\partial}{\partial \boldsymbol{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} -\dfrac{1}{4\pi\varepsilon}\dfrac{Q}{r} & \\ & 0 \end{bmatrix}^{+} \frac{e}{\mathrm{c}} {}^{-}\begin{bmatrix} u_t & \\ & \boldsymbol{u} \end{bmatrix}^{-}
$$

$$
= \frac{1}{4\pi\varepsilon} {}^{-}\begin{bmatrix} -\dfrac{\partial}{\partial \mathrm{c}t}\left(\dfrac{Q}{r}\right) & \\ & \dfrac{\partial}{\partial \boldsymbol{r}}\left(\dfrac{Q}{r}\right) \end{bmatrix}^{+} \frac{e}{\mathrm{c}} {}^{-}\begin{bmatrix} u_t & \\ & \boldsymbol{u} \end{bmatrix}^{-}
$$

$$
= \frac{e}{4\pi\varepsilon \mathrm{c}} {}^{-}\begin{bmatrix} -\dfrac{\partial}{\partial \mathrm{c}t}\left(\dfrac{Q}{r}\right)u_t + \dfrac{\partial}{\partial \boldsymbol{r}}\left(\dfrac{Q}{r}\right)\bullet \boldsymbol{u} & \\ & -\dfrac{\partial}{\partial \mathrm{c}t}\left(\dfrac{Q}{r}\right)\boldsymbol{u} + \dfrac{\partial}{\partial \boldsymbol{r}}\left(\dfrac{Q}{r}\right)u_t - \mathrm{i}\dfrac{\partial}{\partial \boldsymbol{r}}\left(\dfrac{Q}{r}\right)\times \boldsymbol{u} \end{bmatrix}^{-} . \quad (1.1.2)
$$

We can rewrite the coordinate (t,x,y,z) by the coordinate (t,r,θ,φ).

$$
\begin{cases} t = t \\ x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases} .
$$

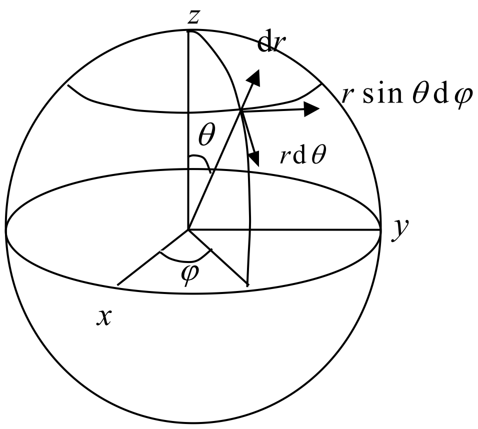


Fig. 1 The spherical coordinate.

And we take the new coordinate (t,r,Ω,φ) by replacing $\theta = \dfrac{\pi}{2} - \mathrm{i}\Omega$ where Ω is the parameter of rotation on the orbit. Then we get the equations of Kepler's type.

1.2 The system of equations of Kepler's type.

The metric is $\mathrm{d}s^2 = -\mathrm{c}^2\mathrm{d}t^2 + \mathrm{d}r^2 + r^2(\cosh^2\Omega\mathrm{d}\varphi^2 - \mathrm{d}\Omega^2)$.

$$
\begin{cases}
m_e c\dfrac{\mathrm{d}ct}{\mathrm{d}\tau} = m_e c C_0 \mathrm{e}^{\frac{keQ}{m_e c^2 r}} \quad \text{(the conservation of energy)} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1.2.1) \\
\dfrac{\mathrm{d}^2}{\mathrm{d}\tau^2}(r\sinh\Omega) = -(\dfrac{keQ}{m_e c^2 r^2}\dfrac{\mathrm{d}ct}{\mathrm{d}\tau})(\tanh\Omega - r\cosh\Omega\dfrac{\mathrm{d}\varphi}{\mathrm{d}ct})\cosh\Omega(\dfrac{\mathrm{d}ct}{\mathrm{d}\tau}) \\
\qquad\qquad\qquad\qquad\qquad\qquad \text{(the structure of space)} \cdots\cdots\cdots(1.2.2) \\
r^2\{(r\cosh\Omega\dfrac{\mathrm{d}\varphi}{\mathrm{d}\tau})^2 - (r\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau})^2\} = C^2 \quad \text{(the law of equal areas)} \cdots\cdots\cdots\cdots\cdots(1.2.3\ ) \\
r^2\cosh\Omega\dfrac{\mathrm{d}\varphi}{\mathrm{d}\tau} = C\cosh\Theta'(\geqq 0), \quad r^2\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau} = -C\sinh\Theta' \\
\quad \Theta' = -\displaystyle\int(\dfrac{keQ}{m_e c^2 r^2}\dfrac{\mathrm{d}ct}{\mathrm{d}\tau} - \sinh\Omega\dfrac{\mathrm{d}\varphi}{\mathrm{d}\tau})d\tau \quad \text{(the internal rotation)} \cdots\cdots\cdots\cdots(1.2.4)
\end{cases}
$$

We put the angular velocity $\dfrac{\mathrm{d}\Phi}{\mathrm{d}\tau} = \sqrt{(\cosh\Omega\dfrac{\mathrm{d}\varphi}{\mathrm{d}\tau})^2 - (\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau})^2}$,the orbit speed $r\dfrac{\mathrm{d}\Phi}{\mathrm{d}\tau}$ and the main equation

$$
(\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\Phi})^2 = (\frac{\mathrm{d}r}{r^2\mathrm{d}\Phi})^2 = -\frac{c^2}{C^2} + \frac{C_0^{\ 2}}{C^2}\mathrm{e}^{\frac{2R_0}{r}} - \frac{1}{r^2}, \quad R_0 = \frac{keQ}{m_e c^2} \fallingdotseq 2.818\times 10^{-15}\mathrm{m}\ .
$$

We call C_0 , C a kinetic energy constant and an equal areas constant respectively. And $\dfrac{R_0}{r}$ is a potential , $\mathrm{e}^{\frac{R_0}{r}}$ is a extended potential.

2.The orbit of the electron in the hydrogen atom.

We differentiate the relation (1.2.1) $\dfrac{\mathrm{d}ct}{\mathrm{d}\tau} = C_0\mathrm{e}^{\frac{R_0}{r}}$ by the orbit angle Φ , $\mathrm{d}\Phi = \sqrt{(\cosh\Omega\mathrm{d}\phi)^2 - (\mathrm{d}\Omega)^2}$. Then we get the relation to the extended potential $\mathrm{e}^{\frac{R_0}{r}}$ and the potential $\dfrac{R_0}{r}$ of space and its tangent space as follows:

$$
\frac{\mathrm{d}}{\mathrm{d}\Phi}(\frac{\mathrm{d}ct}{\mathrm{d}\tau}) = \frac{\mathrm{d}}{\mathrm{d}\Phi}(C_0\mathrm{e}^{\frac{R_0}{r}}) = C_0\frac{\mathrm{e}^{\frac{R_0}{r}}\mathrm{d}\frac{R_0}{r}}{\mathrm{d}\Phi} = C_0\frac{\mathrm{d}\frac{R_0}{r}}{\mathrm{e}^{-\frac{R_0}{r}}\mathrm{d}\Phi}\ .
$$

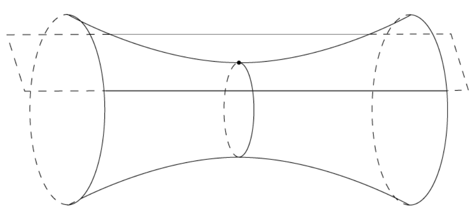


Fig. 2 Anti de-Sitter space.

Therefore, we get the main equation in the Anti de-Sitter space as follows:

$$\left(\frac{\mathrm{d}e^{\frac{R_0}{r}}}{\mathrm{d}\Phi}\right)^2=\left(\frac{R_0\mathrm{d}\frac{1}{r}}{e^{-\frac{R_0}{r}}\mathrm{d}\Phi}\right)^2=R_0{}^2\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{\frac{2R_0}{r}}-\frac{1}{r^2}\right)e^{\frac{2R_0}{r}}\quad\cdot\cdot\ (A).$$

We differentiate this main equation (A), then

$$2\left(\frac{R_0\mathrm{d}\frac{1}{r}}{e^{-\frac{R_0}{r}}\mathrm{d}\Phi}\right)\frac{\mathrm{d}}{\mathrm{d}\Phi}\left(\frac{R_0\mathrm{d}\frac{1}{r}}{e^{-\frac{R_0}{r}}\mathrm{d}\Phi}\right)$$

$$=R_0{}^2\left\{\left(2R_0\frac{C_0{}^2}{C^2}e^{\frac{2R_0}{r}}-\frac{2}{r}\right)e^{\frac{2R_0}{r}}+\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{\frac{2R_0}{r}}-\frac{1}{r^2}\right)2R_0e^{\frac{2R_0}{r}}\right\}\left(\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\Phi}\right).$$

We put $\mathrm{d}\Phi'=e^{-\frac{R_0}{r}}\mathrm{d}\Phi$,then $\dfrac{\mathrm{d}^2\frac{1}{r}}{\mathrm{d}\Phi'^2}=\dfrac{\mathrm{d}}{e^{-\frac{R_0}{r}}\mathrm{d}\Phi}\left(\dfrac{\mathrm{d}\frac{1}{r}}{e^{-\frac{R_0}{r}}\mathrm{d}\Phi}\right)$.

Therefore

$$\begin{aligned}\frac{\mathrm{d}^2\frac{1}{r}}{\mathrm{d}\Phi'^2}&=\left(R_0\frac{C_0{}^2}{C^2}e^{\frac{2R_0}{r}}-\frac{1}{r}\right)e^{\frac{2R_0}{r}}+\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{\frac{2R_0}{r}}-\frac{1}{r^2}\right)R_0e^{\frac{2R_0}{r}}\\&=-\left(R_0\frac{c^2}{C^2}+\frac{1}{r}+R_0\frac{1}{r^2}\right)e^{\frac{2R_0}{r}}+2R_0\frac{C_0{}^2}{C^2}e^{\frac{4R_0}{r}},\quad e^{\frac{2R_0}{r}}=1+2\frac{R_0}{r}+2\frac{R_0{}^2}{r^2}+\cdots\\&=-\frac{1}{2R_0}\left(2R_0{}^2\frac{c^2}{C^2}+2\frac{R_0}{r}+2\frac{R_0{}^2}{r^2}\right)\left(1+2\frac{R_0}{r}+2\frac{R_0{}^2}{r^2}+\cdots\right)\\&\qquad+2R_0\frac{C_0{}^2}{C^2}\left(1+4\frac{R_0}{r}+8\frac{R_0{}^2}{r^2}+\cdots\right)\\&\fallingdotseq-\frac{1}{2R_0}\left\{\left(2R_0{}^2\frac{c^2}{C^2}+2\frac{R_0}{r}\right)+\left(2R_0{}^2\frac{c^2}{C^2}\right)\frac{2R_0}{r}\right\}+2R_0\frac{C_0{}^2}{C^2}\left(1+4\frac{R_0}{r}\right)\\&\fallingdotseq\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)R_0+\left\{\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)R_0{}^2-1\right\}\frac{1}{r}.\end{aligned}$$

This solution is an elliptic curve

$$\frac{1}{r}=\frac{\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)R_0}{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)R_0{}^2}\left(1+e_0\cos\left(\sqrt{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)R_0{}^2}\cdot\Phi'\right)\right).$$

Where e_0 is a constant of integration (an eccentricity).

(i) In the case of the circle orbit($r=r_0$ is constant).

The main equation (A) is

$$(\frac{\mathrm{d}\frac{1}{r}}{\mathrm{d}\Phi'})^2 = (-\frac{\mathrm{c}^2}{C^2} + \frac{C_0^{\ 2}}{C^2}\mathrm{e}^{\frac{2R_0}{r}} - \frac{1}{r^2})\mathrm{e}^{\frac{2R_0}{r}} \equiv 0$$. And

$$\frac{\mathrm{d}^2\frac{1}{r}}{\mathrm{d}\Phi'^2} = (R_0\frac{C_0^{\ 2}}{C^2}\mathrm{e}^{\frac{2R_0}{r}} - \frac{1}{r})\mathrm{e}^{\frac{2R_0}{r}} + (-\frac{\mathrm{c}^2}{C^2} + \frac{C_0^{\ 2}}{C^2}\mathrm{e}^{\frac{2R_0}{r}} - \frac{1}{r^2})R_0\mathrm{e}^{\frac{2R_0}{r}} \equiv 0 .$$

Then

$\frac{1}{r^2} = -\frac{\mathrm{c}^2}{C^2} + \frac{C_0^{\ 2}}{C^2}\mathrm{e}^{\frac{2R_0}{r}}$ and $\frac{1}{r} = R_0\frac{C_0^{\ 2}}{C^2}\mathrm{e}^{\frac{2R_0}{r}}$, therefore $\frac{1}{r^2} = -\frac{\mathrm{c}^2}{C^2} + \frac{1}{rR_0}$

$$\therefore \frac{\mathrm{c}^2}{C^2}r^2 - \frac{1}{R_0}r + 1 = 0 \quad \therefore \frac{\mathrm{c}^2}{(r\frac{\mathbf{v}}{\sqrt{1-(\frac{\mathbf{v}}{\mathrm{c}})^2}})^2}r^2 - \frac{1}{R_0}r + 1 = 0 \quad \therefore (\frac{\mathbf{v}}{\mathrm{c}})^2 = \frac{R_0}{r} \quad \cdot\cdot\ \text{(B)}.$$

This relation is the balance equation of the centrifugal force and the attractive force. $\frac{m_e\mathbf{v}^2}{r} = \frac{m_e\mathrm{c}^2R_0}{r^2} = \frac{keQ}{r^2}$, $R_0 = \frac{keQ}{m_e\mathrm{c}^2}$.

Moreover,

$$r = \frac{\frac{1}{R_0} \pm \sqrt{\frac{1}{R_0^{\ 2}} - 4\frac{\mathrm{c}^2}{C^2}}}{2\frac{\mathrm{c}^2}{C^2}}, \quad 4\frac{\mathrm{c}^2}{C^2} < \frac{1}{R_0^{\ 2}} \quad \text{(from the discriminant)}.$$

Then $\frac{\mathrm{c}}{C}[= \frac{\mathrm{c}}{r\frac{\mathbf{v}}{\sqrt{1-(\frac{\mathbf{v}}{\mathrm{c}})^2}}}] = \frac{1}{2R_0}$.

Therefore, the minimum of this radius[6] and the maximum velocity are

$r = 2R_0 = r_s$ and $\mathbf{v} = \frac{\mathrm{c}}{\sqrt{2}}$.

(ii) In the case of the parabolic orbit.

At the infinity, $r = \infty$, $\mathbf{v} = 0$. Then by the relation (1.2.1) $\frac{\mathrm{c}}{\sqrt{1-(\frac{\mathbf{v}}{\mathrm{c}})^2}}(= \frac{\mathrm{d}\mathrm{c}t}{\mathrm{d}\tau}) = C_0\mathrm{e}^{\frac{R_0}{r}}$, we get $C_0 = \mathrm{c}$. Therefore

$$\frac{\mathrm{d}^2 \frac{1}{r}}{\mathrm{d}\Phi'^2} = \frac{\mathrm{c}^2}{C^2} R_0 + (6\frac{\mathrm{c}^2}{C^2} R_0{}^2 - 1)\frac{1}{r}.$$

This solution $\frac{1}{r} = \frac{\frac{\mathrm{c}^2}{C^2} R_0}{1 - 6\frac{\mathrm{c}^2}{C^2} R_0{}^2}(1 - e_0 \cos\left(\sqrt{1 - 6\frac{\mathrm{c}^2}{C^2} R_0{}^2} \cdot \Phi'\right))$, $e_0 = 1$ is a parabola.

3. Example : The light emitting mechanism.

(i) The elliptic and the circle orbits.

When the electron in the elliptic orbit with the kinetic energy constant $C_0 = \frac{\mathrm{c}e^{-\frac{R_0}{r}}}{\sqrt{1 - (\frac{\mathbf{v}}{\mathrm{c}})^2}}$ and the equal areas constant $C = r^2 \frac{\mathrm{d}\Phi}{\mathrm{d}\tau}$,

$\frac{\mathrm{d}\Phi}{\mathrm{d}\tau} = \sqrt{(\cosh\Omega \frac{\mathrm{d}\varphi}{\mathrm{d}\tau})^2 - (\frac{\mathrm{d}\Omega}{\mathrm{d}\tau})^2}$ has the speed $\mathbf{v}_0$ at the perihelion r_0 (the distance), we get

$$C_0 = \frac{\mathrm{c}e^{-\frac{R_0}{r_0}}}{\sqrt{1 - (\frac{\mathbf{v}_0}{\mathrm{c}})^2}}, \quad C = r_0 \frac{\mathbf{v}_0}{\sqrt{1 - (\frac{\mathbf{v}_0}{\mathrm{c}})^2}}, \quad \mathbf{v}_0 = r_0 \frac{\mathrm{d}\Phi}{\mathrm{d}t} = r_0 \cosh\Omega \frac{\mathrm{d}\varphi}{\mathrm{d}t}, \quad \frac{\mathrm{d}\Omega}{\mathrm{d}\tau} = 0.$$

Especially, when the circle orbit O_m with radius r_m for a number m orbit, we get

$$C_m (= C_0) = \frac{\mathrm{c}e^{-\frac{R_0}{r_m}}}{\sqrt{1 - \frac{R_0}{r_m}}}, \quad E_m (= C) = r_m \frac{\mathrm{c}\sqrt{\frac{R_0}{r_m}}}{\sqrt{1 - \frac{R_0}{r_m}}}$$ from the relation (B) $(\frac{\mathbf{v}}{\mathrm{c}})^2 = \frac{R_0}{r}$.

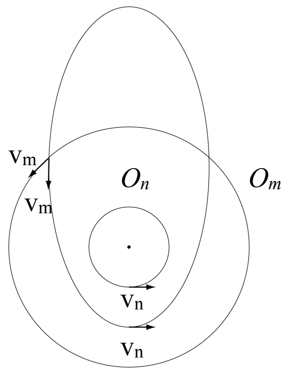
(ii) The specific elliptic orbit.

The electron in the elliptic orbit which has the same kinetic energy constant C_m in the circle orbit O_m has the speed $\mathbf{v}$ at any point r so that

$$\frac{\mathrm{c}e^{-\frac{R_0}{r_m}}}{\sqrt{1-\frac{R_0}{r_m}}}(=C_m)=\frac{\mathrm{c}e^{-\frac{R_0}{r}}}{\sqrt{1-(\frac{\mathbf{v}}{\mathrm{c}})^2}}\quad \therefore (\frac{\mathbf{v}}{\mathrm{c}})^2=1-(1-\frac{R_0}{r_m})e^{\frac{2R_0}{r_m}-\frac{2R_0}{r}} \fallingdotseq 2\frac{R_0}{r}-\frac{R_0}{r_m}.$$

When this electron has the same speed $\mathbf{v}_n(=\mathbf{v}_0)$ at the perihelion r_0 which speed $\mathbf{v}_n$ is the rounding speed for an electron in another circle orbit O_n ($r_n < r_m$), then

$$\frac{\mathrm{c}e^{-\frac{R_0}{r_m}}}{\sqrt{1-\frac{R_0}{r_m}}}(=C_m)=\frac{\mathrm{c}e^{-\frac{R_0}{r_0}}}{\sqrt{1-(\frac{\mathbf{v}_n}{\mathrm{c}})^2}}=\frac{\mathrm{c}e^{-\frac{R_0}{r_0}}}{\sqrt{1-\frac{R_0}{r_n}}}.$$



And we put its radius $r_{[m\to n]}(=r_0)$,then

Fig. 3 Specific elliptic orbit.

$$e^{\frac{R_0}{r_{[m\to n]}}}(=e^{\frac{R_0}{r_0}})=\frac{\sqrt{1-\frac{R_0}{r_m}}}{\sqrt{1-\frac{R_0}{r_n}}}e^{\frac{R_0}{r_m}}\quad \therefore \frac{2}{r_{[m\to n]}}\fallingdotseq\frac{1}{r_n}+\frac{1}{r_m}\quad \cdot\cdot\ \text{At the same speed point } r_{[m\to n]}.$$

(iii) The track change and the radiation of light.

We considered that "The light is radiated" to be "the electromagnetic field changes for a moment". We define the acceleration quantity which is a new idea, and can create the mechanism by which light is radiated.

The 4-dimensional force is the differential of the 4-dimensional moment as follows:

$${}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-}=m_e\frac{\mathrm{d}}{\mathrm{d}\tau}\,{}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-}.$$

On the other hand, the 4-dimensional force on the moving charge is Coulomb-Lorentz force and is represented as follows:

$${}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-}={}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-\mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+}\frac{e}{\mathrm{c}}\,{}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-}.$$

Therefore $m_e\frac{\mathrm{d}}{\mathrm{d}\tau}\,{}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-}=\frac{e}{\mathrm{c}}\,{}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-\mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+}\,{}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-}$ (The relation in space).

We recognize another meaning that the electromagnetic field causes the change of the state of moving charge. Therefore by the relation ${}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-}\,{}^{+}\begin{bmatrix} u_0 & \\ & -\boldsymbol{u} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} u_0{}^2 - \boldsymbol{u}^2 & \\ & 0 \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \mathrm{c}^2 & \\ & 0 \end{bmatrix}^{+}$,

$$
{}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E} - \mathrm{i}\mathrm{c}\boldsymbol{B} \end{bmatrix}^{+} = \frac{m_e \mathrm{c}}{e} \frac{\mathrm{d}}{\mathrm{d}\tau}\, {}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-} \cdot {}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-,-1}
$$

$$
= \frac{m_e \mathrm{c}}{e} \frac{\mathrm{d}}{\mathrm{d}\tau}\, {}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-} \cdot \frac{1}{\mathrm{c}^2}\, {}^{+}\begin{bmatrix} u_0 & \\ & -\boldsymbol{u} \end{bmatrix}^{+} \quad \text{(The relation in tangent space).}
$$

We call this physical quantity the acceleration quantity which is another name and the meaning of electromagnetic field. We assumed that the electron changes the track from the elliptic orbit to the circle orbit at this perihelion $r_{[m\to n]}$ and considered in this situation. And the result of piling up this acceleration quantity causes the electron to change the track. Vice versa, this track change brings about the braking radiation. We go into more detail about this way, the energy between the orbit O_m and the orbit O_n is the difference of energy at the same speed points, that is,

$$
\Delta E_{m\to n} = \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_m}}}{\sqrt{1-\frac{R_0}{r_m}}} - \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_n}}}{\sqrt{1-\frac{R_0}{r_n}}} = \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_{[m\to n]}}}}{\sqrt{1-\frac{R_0}{r_n}}} - \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_n}}}{\sqrt{1-\frac{R_0}{r_n}}}, \quad \frac{2}{r_{[m\to n]}} \doteqdot \frac{1}{r_n} + \frac{1}{r_m}
$$

$$
\doteqdot \frac{m_e \mathrm{c}^2}{\sqrt{1-\frac{R_0}{r_n}}} \left(\frac{R_0}{r_n} - \frac{R_0}{r_{[m\to n]}} \right) \cdot\cdot\cdot \text{(C).}
$$

At the points on the circle orbit r_n and the perihelion $r_{[m\to n]}$, the speeds are the same but their surrounding frequency energy[6] ,that is, the states of rotation are not equal. Therefore we pile up the acceleration quantity as conserving the same speed and can change the track of the electron. By the balance equation $\dfrac{m_e \mathbf{v}^2}{r} = \dfrac{m_e \mathrm{c}^2 R_0}{r^2} = \dfrac{keQ}{r^2}$ on the circle orbit, when the charge Q at the center vary from $Q_{[m\to n]} = \dfrac{r_{[m\to n]} m_e \mathbf{v}^2}{ke}$ to $Q_n = \dfrac{r_n m_e \mathbf{v}^2}{ke} (= Q)$, the radius vary from $r_{[m\to n]}$ to r_n with conserving the same speed. Vice versa, when the track changes from $r_{[m\to n]}$ to r_n , the electromagnetic field changes for a moment.

(iv) The frequency of light and the surrounding frequency energy.

"The Planck relation $E = h\nu$ means that the Planck constant h is the proportionality constant between the energy E of a photon (or light) and the frequency ν of it."

As what realizes this phrase, we take the angular momentum

$$h' = 2\pi r_1 m_e \frac{\mathbf{v}_1}{\sqrt{1-(\frac{\mathbf{v}_1}{\mathrm{c}})^2}} = \frac{2\pi r_1 m_e \mathrm{c}\sqrt{\frac{R_0}{r_1}}}{\sqrt{1-\frac{R_0}{r_1}}}$$

in the orbit O_1 instead of Planck constant which is an earlier idea of Planck.

We change the formula (C) as follows:

$$\Delta E_{m\to n} = \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_m}}}{\sqrt{1-\frac{R_0}{r_m}}} - \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_n}}}{\sqrt{1-\frac{R_0}{r_n}}} = (m_e \mathrm{c}^2 - \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_n}}}{\sqrt{1-\frac{R_0}{r_n}}}) - (m_e \mathrm{c}^2 - \frac{m_e \mathrm{c}^2 \mathrm{e}^{-\frac{R_0}{r_m}}}{\sqrt{1-\frac{R_0}{r_m}}})$$

$$= m_e \mathrm{c}^2 \frac{\sqrt{1-\frac{R_0}{r_n}} - \mathrm{e}^{-\frac{R_0}{r_n}}}{\sqrt{1-\frac{R_0}{r_n}}} - m_e \mathrm{c}^2 \frac{\sqrt{1-\frac{R_0}{r_m}} - \mathrm{e}^{-\frac{R_0}{r_m}}}{\sqrt{1-\frac{R_0}{r_m}}}$$

$$\fallingdotseq m_e \mathrm{c}^2 \frac{\frac{R_0}{2r_n}}{\sqrt{1-\frac{R_0}{r_n}}} - m_e \mathrm{c}^2 \frac{\frac{R_0}{2r_m}}{\sqrt{1-\frac{R_0}{r_m}}} = m_e \mathrm{c}^2 \underline{\frac{\frac{R_0}{2r_n}}{\sqrt{1-\frac{R_0}{2r_n}}}} \frac{\sqrt{1-\frac{R_0}{2r_n}}}{\sqrt{1-\frac{R_0}{r_n}}} - m_e \mathrm{c}^2 \underline{\underline{\frac{\frac{R_0}{2r_m}}{\sqrt{1-\frac{R_0}{r_m}}}}} \frac{\sqrt{1-\frac{R_0}{2r_m}}}{\sqrt{1-\frac{R_0}{r_m}}}.$$

This energy is the difference between two surrounding frequency energy[6]

$$E_{[n]}' = \frac{m_e \mathrm{c}^2 (\frac{\mathbf{v}_{[n]}}{\mathrm{c}})^2}{\sqrt{1-(\frac{\mathbf{v}_{[n]}}{\mathrm{c}})^2}} = m_e \mathrm{c}^2 \underline{\frac{\frac{R_0}{2r_n}}{\sqrt{1-\frac{R_0}{2r_n}}}} \quad \text{and} \quad E_{[m]}' = \frac{m_e \mathrm{c}^2 (\frac{\mathbf{v}_{[m]}}{\mathrm{c}})^2}{\sqrt{1-(\frac{\mathbf{v}_{[m]}}{\mathrm{c}})^2}} = m_e \mathrm{c}^2 \underline{\underline{\frac{\frac{R_0}{2r_m}}{\sqrt{1-\frac{R_0}{2r_m}}}}}$$

as an approximation.

Therefore, $\Delta E_{m\to n} \doteqdot E_{[n]}{}' - E_{[m]}{}'$ $\quad \therefore \nu_{m\to n} = \dfrac{\Delta E_{m\to n}}{h'} \doteqdot \dfrac{E_{[n]}{}'}{h'} - \dfrac{E_{[m]}{}'}{h'} = \nu_{[n]} - \nu_{[m]}$.

The frequency and wave length of electron is

$$\nu_{[n]} = \frac{E_{[n]}{}'}{h'} = \frac{m_e \mathrm{c}^2 \dfrac{(\frac{\mathbf{v}_{[n]}}{\mathrm{c}})^2}{\sqrt{1-(\frac{\mathbf{v}_{[n]}}{\mathrm{c}})^2}}}{2\pi r_1 m_e \dfrac{\mathbf{v}_1}{\sqrt{1-(\frac{\mathbf{v}_1}{\mathrm{c}})^2}}} = \frac{\mathbf{v}_{[n]}}{2\pi r_1} \frac{\dfrac{\mathbf{v}_{[n]}}{\sqrt{1-(\frac{\mathbf{v}_{[n]}}{\mathrm{c}})^2}}}{\dfrac{\mathbf{v}_1}{\sqrt{1-(\frac{\mathbf{v}_1}{\mathrm{c}})^2}}},$$

$$\lambda_{[n]} = \frac{h'}{P_{[n]}} = \frac{2\pi r_1 m_e \dfrac{\mathbf{v}_1}{\sqrt{1-(\frac{\mathbf{v}_1}{\mathrm{c}})^2}}}{m_e \mathrm{c} \dfrac{\frac{\mathbf{v}_{[n]}}{\mathrm{c}}}{\sqrt{1-(\frac{\mathbf{v}_{[n]}}{\mathrm{c}})^2}}} = \frac{2\pi r_1 \mathbf{v}_1}{\sqrt{1-(\frac{\mathbf{v}_1}{\mathrm{c}})^2}} \frac{1}{\dfrac{\mathbf{v}_{[n]}}{\sqrt{1-(\frac{\mathbf{v}_{[n]}}{\mathrm{c}})^2}}} \quad \text{and} \quad \lambda_{[n]}\nu_{[n]} = \mathbf{v}_{[n]}.$$

And this frequency serves as a basis of the frequency of light.

Conclusion

The authors can calculate an electron orbit around the proton in atom and generate the light, and we show that the frequency of radiated light is reduced to the surrounding frequency energy of electron. This suggests that we can adapt this method for any other transit electron.

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