~~ミンコフスキー空間のローレンツ変換に関する新しい概念と基本的なツール~~

New Concept and Basic Tools

of the Lorentz Transformation in a Minkowski Space

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Abstract

The purpose of this paper is to show the new concept of the Lorentz transformation, and its application to the electromagnetic theory.

Using the Lorentz transformation, we have a many coordinate systems in Minkowski spaces. However we can treat these coordinate systems only in one coordinate system.

Authors propose the helpful items which are simultaneous reach surface and hyperbolic radian of the situation in a Minkowski space, and then we can get the essence of some events in the 4-dimensional Time-space (the Minkowski space).

In the electromagnetic theory, we have a new phase of a moving charge. Moreover we will have an idea of "stream charge" in place of the electric current.[1],[2]

key words: Lorentz transformation,Minkowski spaces,

Simultaneous reach surface,Hyperbolic radian,Stream charge,En bloc

Contents

1. **Preliminaries**

1.1 The component of a vector and its rotation

For simplicity, we substituted $v_z = 0$ for the velocity vector $\boldsymbol{v} = \boldsymbol{v}\left(v_x, v_y, v_z\right)$. Then the rotation by the angle θ_0 is as follows:

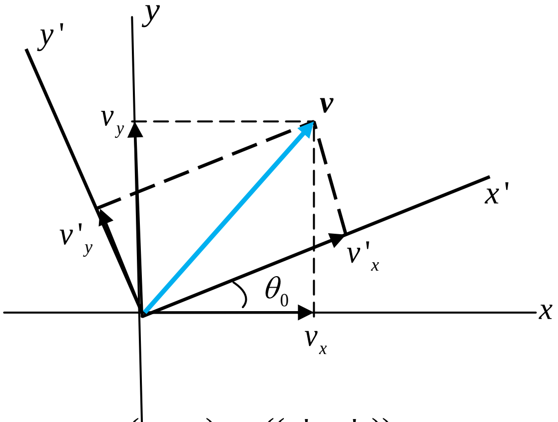

Figure 1. $\boldsymbol{v} = \boldsymbol{v}(v_x, v_y) = \boldsymbol{v}((v'_x, v'_y))$

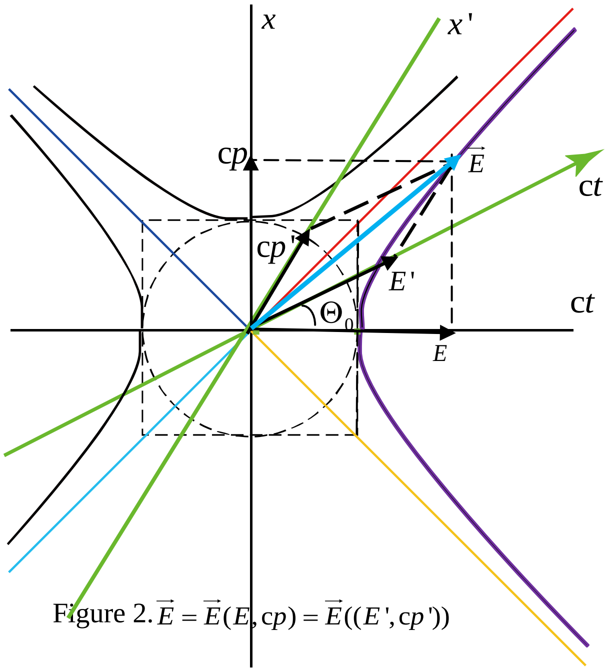

Figure 2. $\vec{E} = \vec{E}(E, cp) = \vec{E}((E', cp'))$

Then the rotation of the component $\boldsymbol{v}(v_x, v_y)$ is

$$\begin{pmatrix} v'_x \\ v'_y \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} \quad \cdot\cdot (*)$$

which is showed by blue arrows in Figure 1. However, the substance v is still unbudging.

Definition. In a Euclid space, a velocity vector is translated as above (*) by the rotation, we call this physical quantity "proper".

1.2 The relation between the rotation and the Lorentz transformation

If we use two axes $\boldsymbol{e}_1 = e^{i\alpha} ct$, $\boldsymbol{e}_2 = e^{i\beta} x$ and angle θ_0, then the coordinate $\begin{pmatrix} v_{ct} \\ v_x \end{pmatrix}$ of vector $\boldsymbol{v} = (\boldsymbol{e}_1, \boldsymbol{e}_2)\begin{pmatrix} v_{ct} \\ v_x \end{pmatrix}$ is changed into the coordinate $\begin{pmatrix} v'_{ct} \\ v'_x \end{pmatrix}$ of the vector $\boldsymbol{v} = (\boldsymbol{e}'_1, \boldsymbol{e}'_2)\begin{pmatrix} v'_{ct} \\ v'_x \end{pmatrix}$ by the rotation θ_0.

Then its coordinate transformation is $\begin{pmatrix} v'_{ct} \\ v'_x \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} v_{ct} \\ v_x \end{pmatrix}$. By this transformation, the substance of $\boldsymbol{v}(v_{ct}, v_x)$ is unbudging.

We put $\alpha = 0$, $\beta = \dfrac{\pi}{2}$ in the coordinate transformation, then the axes become $\boldsymbol{e}_1 = \mathrm{c}t$, $\boldsymbol{e}_2 = \mathrm{i}x$

and the coordinate $\begin{pmatrix} v_{ct} \\ v_x \end{pmatrix}$ of vector $\boldsymbol{v} = (\mathrm{c}t, \mathrm{i}x)\begin{pmatrix} v_{ct} \\ v_x \end{pmatrix} = (\mathrm{c}t, x)\underline{\underline{\begin{pmatrix} v_{ct} \\ \mathrm{i}v_x \end{pmatrix}}}$ is changed into the coordinate

$\begin{pmatrix} v'_{ct} \\ v'_x \end{pmatrix}$ of the vector $\boldsymbol{v}' = (\mathrm{c}t, \mathrm{i}x)\begin{pmatrix} v'_{ct} \\ v'_x \end{pmatrix} = (\mathrm{c}t, x)\underline{\underline{\begin{pmatrix} v'_{ct} \\ \mathrm{i}v'_x \end{pmatrix}}}$.

Therefore $\underline{\underline{\begin{pmatrix} v'_{ct} \\ \mathrm{i}v'_x \end{pmatrix}}} = \begin{pmatrix} \cos\theta_0 & -\mathrm{i}\sin\theta_0 \\ -\mathrm{i}\sin\theta_0 & \cos\theta_0 \end{pmatrix}\begin{pmatrix} v_{ct} \\ \mathrm{i}v_x \end{pmatrix} = \begin{pmatrix} \cosh\Theta_0 & -\sinh\Theta_0 \\ -\sinh\Theta_0 & \cosh\Theta_0 \end{pmatrix}\underline{\underline{\begin{pmatrix} v_{ct} \\ \mathrm{i}v_x \end{pmatrix}}}$, $\theta_0 = -\mathrm{i}\Theta_0$ holds.

This rotation

$$\begin{pmatrix} V_{ct} \\ V_x \end{pmatrix} = \underline{\underline{\begin{pmatrix} v_{ct} \\ \mathrm{i}v_x \end{pmatrix}}} \to \begin{pmatrix} V'_{ct} \\ V'_x \end{pmatrix} = \underline{\underline{\begin{pmatrix} v'_{ct} \\ \mathrm{i}v'_x \end{pmatrix}}} = \begin{pmatrix} \cosh\Theta_0 & -\sinh\Theta_0 \\ -\sinh\Theta_0 & \cosh\Theta_0 \end{pmatrix}\begin{pmatrix} V_{ct} \\ V_x \end{pmatrix} \quad \cdot\cdot\,(**)$$

is a Lorentz transformation when the particle moves to the x−direction with the speed $\dfrac{v}{\mathrm{c}} = \tanh\Theta_0$

but, the meaning of this angle $\Theta_0 = \mathrm{i}\theta_0$ is unknown.

Definition. <u>In a Minkowski space, a velocity vector is translated as above (**) by the Lorentz transformation, we call this physical quantity "proper".</u>

Example 1 (The 4-dimensional momentum).

When a particle is stable, then its energy E is $m\mathrm{c}^2$ and the momentum is zero. Therefore, when the particle is moving with the velocity $\boldsymbol{v}$ for the x-direction, its component is $\boldsymbol{v}(\gamma, \gamma\beta_x, 0, 0)$ by use of the 4-dimensional velocity. Therefore we can put the 4-dimensional momentum which is showed by blue arrows in Figure 2.

$$\vec{E} = \begin{bmatrix} E & \\ & \mathrm{c}\boldsymbol{p} \end{bmatrix} = \begin{bmatrix} m\mathrm{c}^2\gamma & \\ & m\mathrm{c}^2\gamma\beta \end{bmatrix},$$

Where $E = m\mathrm{c}^2\gamma$ is a energy and $\boldsymbol{p} = m\mathrm{c}\gamma\beta$ is a momentum

$$E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\sim mc^2+\frac{1}{2}mv^2+\bullet\bullet\bullet\text{(Energy)},$$

$$\boldsymbol{p}=\frac{m\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}\sim m\boldsymbol{v}+\frac{m\boldsymbol{v}^3}{2c^2}+\bullet\bullet\bullet\text{ (Momentum)}.$$

And $\gamma=\dfrac{1}{\sqrt{1-(\frac{v_x}{c})^2}}=\dfrac{u_0}{c}=\cosh\Theta$, $\gamma\beta_x=\dfrac{\frac{v_x}{c}}{\sqrt{1-(\frac{v_x}{c})^2}}=\dfrac{u_x}{c}=\sinh\Theta$,

Furthermore, a part of signatures "-" represent the invariance of the Lorentz transformation, i.e. means the contravariance of vectors. We call this expression matrix vector and Lorentz form.
The proper physical quantity (4-dimensional momentum) is represented by the same point in the Minkowski space even if the component is changed by the Lorentz transformation.

The Lorentz transformation is expressed by using the matrix vector and Lorentz form as follows;

$${}^{-}\begin{bmatrix}\frac{E}{c} & \\ & \boldsymbol{p}\end{bmatrix}^{-}=\begin{bmatrix}\gamma_+ & \\ & -\boldsymbol{\gamma}_-\end{bmatrix}\begin{bmatrix}\frac{E}{c} & \\ & \boldsymbol{p}\end{bmatrix}\begin{bmatrix}\gamma_+ & \\ & -\boldsymbol{\gamma}_-\end{bmatrix},\gamma_+=\cosh\frac{\Theta}{2},\boldsymbol{\gamma}_-=(\sinh\frac{\Theta}{2},0,0),$$

where $\dfrac{u_0}{c}=\gamma=\cosh\Theta$, $\dfrac{u_x}{c}=\gamma\beta_x=\sinh\Theta$.

2. The new items

Theorem 1.

In Figure 3, the angle $\Theta = \mathrm{i}\theta$ is the arc length from A to B in the simultaneous reach line. We named this angle Θ a hyperbolic radian.

(Proof)

In general, the arc length of the circle $(x, y) = r_0(\cos\theta, \sin\theta)$ is

$$L = \int_0^{\theta} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta = r_0 \int_0^{\theta} \sqrt{\sin^2\theta + \cos^2\theta}\,\mathrm{d}\theta = r_0 \int_0^{\theta} d\theta = r_0\theta\,.$$

We use the same technique in the Figure 3,

and the velocity $\dfrac{\mathrm{d}x}{\mathrm{dct}} = \tanh\Theta$ of a particle is constant.

Then the formula $\dfrac{\mathrm{d}ct}{\mathrm{d}\tau} = u_0 = \mathrm{c}\cosh\Theta$,

and $\dfrac{\mathrm{d}x}{\mathrm{d}\tau} = u_x = \mathrm{c}\sinh\Theta$ holds.

When the proper time τ_0 passes,

then the particles reach

$A_1(\mathrm{ct}, x) = A_1(\mathrm{c}\tau_0\cosh\Theta, \mathrm{c}\tau_0\sinh\Theta)$

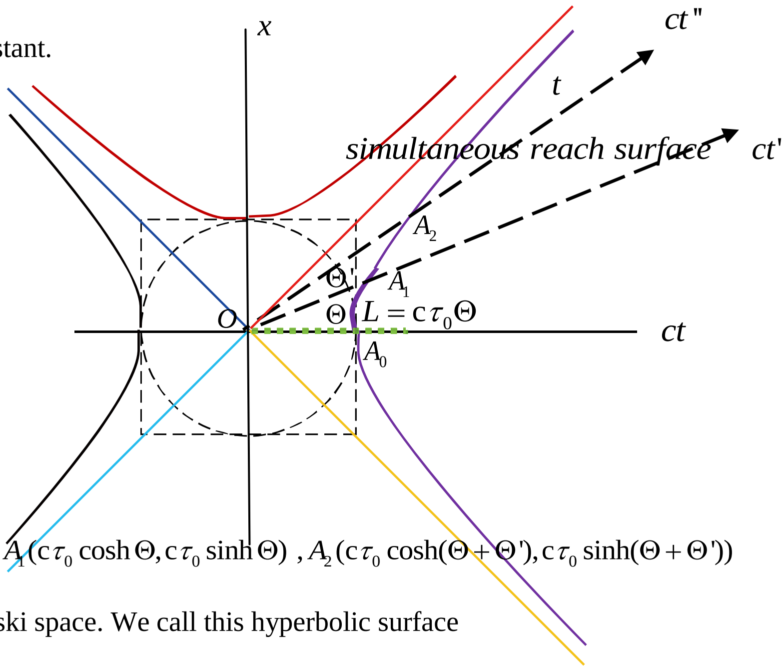

Figure 3. $A_0(\mathrm{c}\tau_0, 0)$, $A_1(\mathrm{c}\tau_0\cosh\Theta, \mathrm{c}\tau_0\sinh\Theta)$, $A_2(\mathrm{c}\tau_0\cosh(\Theta+\Theta'), \mathrm{c}\tau_0\sinh(\Theta+\Theta'))$

and this point is in the surface $(\mathrm{c}t)^2 - x^2 = (\mathrm{c}\tau_0)^2$ in the Minkowski space. We call this hyperbolic surface simultaneous reach surface.

Thus the arc length L on the hyperbolic line from the point $A_0(\mathrm{c}\tau_0, 0)$ to the point A_1 is

$$L = \int_0^{\Theta} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\Theta}\right)^2 - \left(\frac{\mathrm{d}ct}{\mathrm{d}\Theta}\right)^2}\,\mathrm{d}\Theta = \mathrm{c}\tau_0 \int_0^{\Theta} \sqrt{\cosh^2\Theta - \sinh^2\Theta}\,\mathrm{d}\Theta = \mathrm{c}\tau_0 \int_0^{\Theta} \mathrm{d}\Theta = \mathrm{c}\tau_0\Theta.$$

Therefore we can define the angle $\Theta(= -\mathrm{i}\theta)$ as the same arc length Θ.

(Q.E.D.)

Corollary

This arc length Θ and the simultaneous line (or surface) are new items and admissible to the Lorentz transformation.

(1)Any two points in the simultaneous line are shiftable each other by the Lorentz transformation.

(2)The composition of speed is the sum of the arc length.

(Proof)

(1) When two points $(ct,x)=(c\tau\cosh\Theta, c\tau\sinh\Theta)$, $(ct',x')=(c\tau\cosh(\Theta+\Theta'), c\tau\sinh(\Theta+\Theta'))$ are on the simultaneous reach line in the future.

Then

$$
\begin{aligned}
(ct',x') &= (c\tau\cosh(\Theta+\Theta'), c\tau\sinh(\Theta+\Theta')) \\
&= c\tau(\cosh\Theta\cosh\Theta'+\sinh\Theta\sinh\Theta', \cosh\Theta\sinh\Theta'+\sinh\Theta\cosh\Theta') \\
&= (ct\cosh\Theta'+x\sinh\Theta', ct\sinh\Theta'+x\cosh\Theta').
\end{aligned}
$$

(2) The case of two speeds $\dfrac{\boldsymbol{v}}{c}=\tanh\Theta, \dfrac{\boldsymbol{v}'}{c}=\tanh\Theta'$, then the composition of these speeds is $\tanh(\Theta+\Theta')$. This is the speed which corresponds to the sum of the arc length.

(Q.E.D.)

Example 2 (Time dilation and Length contraction)**.**

<u>The 4-dimensional velocity (proper physical quantity) is represented by the same point in the Minkowski space even if the component is changed by the Lorentz transformation.</u>

(i)The Lorentz transformation in the Minkowski space is represented as the point in the simultaneous surface in Figure 4.

We put the point $A_0(c\tau_0, 0)$.

Then the point $B(c\tau_0\cosh\Theta, 0)$ means the "time dilation".

(ii)We differentiate the equation of the same distance surface $(x)^2+(y)^2+(z)^2-(ct)^2=1$, then we get $2xdx+2ydy+2zdz-2ctdct=0$..

Therefore $OC_1\perp C_1E$ (the orthogonal) and $C_1E \parallel OA_1$ (the parallel).

We put the point $C_0(0,x_0)$ and calculate the point $E(0,X)$ as $\dfrac{DE}{DC_1}=\dfrac{x_0\cosh\Theta - X}{x_0\sinh\Theta}$ is the leaning $\tanh\Theta$,

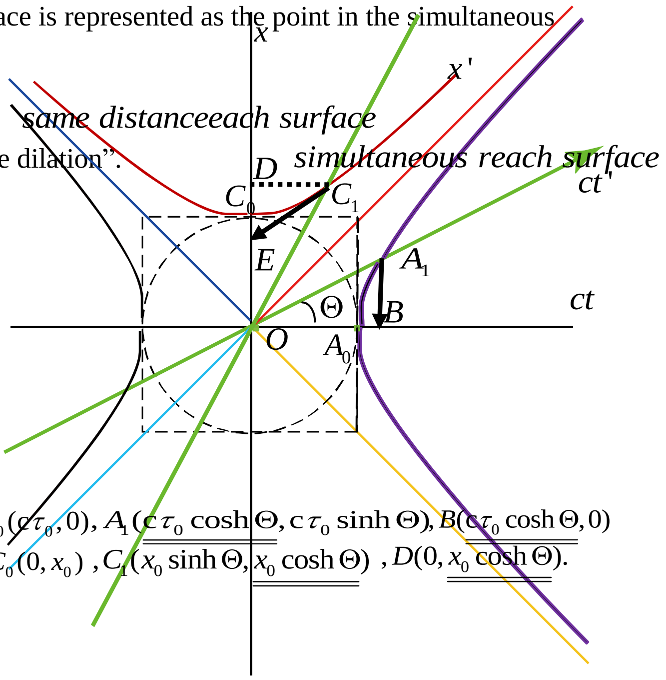

Figure 4. $A_0(c\tau_0,0)$, $A_1(c\tau_0\cosh\Theta, c\tau_0\sinh\Theta)$, $B(c\tau_0\cosh\Theta,0)$, $C_0(0,x_0)$, $C_1(x_0\sinh\Theta, x_0\cosh\Theta)$, $D(0,x_0\cosh\Theta)$.

$$\frac{x_0 \sinh^2 \Theta}{\cosh \Theta} = x_0 \cosh \Theta - X \quad \therefore X = x_0 \cosh \Theta - \frac{x_0 \sinh^2 \Theta}{\cosh \Theta} = \frac{x_0}{\cosh \Theta}.$$

Therefore the point E is $(0, \frac{x_0}{\cosh \Theta})$. This means the " length contraction".

3. The application to the electromagnetic theory

The moving charge q has an expression as a 4-dimensional matrix vector.

When its velocity is $\boldsymbol{v} = \boldsymbol{v}(v_x, v_y, v_z)$, then $\vec{q} = {}^{-}\begin{bmatrix} q\gamma & \\ & q\gamma\beta \end{bmatrix}^{-} = \frac{q}{\mathrm{c}} {}^{-}\begin{bmatrix} u_0 & \\ & \boldsymbol{u} \end{bmatrix}^{-}$, we call the $q\gamma$ a the relativistic charge , $\frac{j_s}{\mathrm{c}} = q\gamma\beta$ a stream charge and $\vec{q}$ "en bloc" which means all together.

Its covariant vector is ${}^{+}\begin{bmatrix} q\gamma & \\ & -q\gamma\beta \end{bmatrix}^{+}$, a pair of signatures "+" represent the invariance of the Lorentz transformation.

Example 3

There are two charges q at the origin and q' at the point $(x, y, 0)$ and the distance is $|\boldsymbol{r}| = \sqrt{x^2 + y^2}$ in Figure 5.

(i)The potential of q is ${}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\boldsymbol{A} \end{bmatrix}^{+} = \frac{1}{4\pi\varepsilon_0} {}^{+}\begin{bmatrix} \frac{q}{r} & \\ & \mathbf{0} \end{bmatrix}^{+}$ which is covariant vector.

The field is

$${}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E} - \mathrm{i}\mathrm{c}\boldsymbol{B} \end{bmatrix}^{+} = \frac{1}{4\pi\varepsilon_0} {}^{-}\begin{bmatrix} \frac{\partial}{\partial \mathrm{c}t} & \\ & -\frac{\partial}{\partial \boldsymbol{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \frac{q}{r} & \\ & \mathbf{0} \end{bmatrix}^{+} = \frac{q}{4\pi\varepsilon_0 r^3} {}^{-}\begin{bmatrix} 0 & \\ & (x, y, z) \end{bmatrix}^{+}.$$

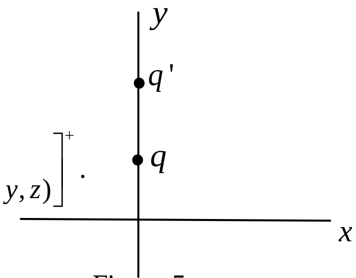

Figure 5.

The force at the point $(x, y, 0)$ is

$${}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = \frac{q}{4\pi\varepsilon_0 r^3} {}^{-}\begin{bmatrix} 0 & \\ & (x, y, 0) \end{bmatrix}^{+} {}^{-}\begin{bmatrix} q' & \\ & \mathbf{0} \end{bmatrix}^{-} = \frac{qq'}{4\pi\varepsilon_0 r_0^{\,2}} {}^{-}\begin{bmatrix} 0 & \\ & (x, y, 0) \end{bmatrix}^{-}.$$

(ii) When the two charges move $\mathbf{v} = \mathbf{v}(v_x, 0, 0)$ in Figure 6.

The potential is ${}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\boldsymbol{A} \end{bmatrix}^{+} = \frac{1}{4\pi\varepsilon_0} {}^{+}\begin{bmatrix} \frac{q\gamma}{r} & \\ & (-\frac{q\gamma\beta_x}{r}, 0, 0) \end{bmatrix}^{+}$.

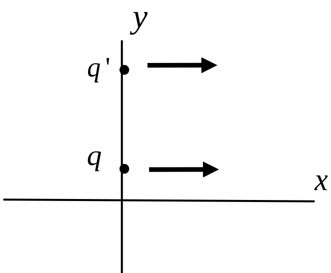

Figure 6.

The field is ${}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E} - \mathrm{ic}\boldsymbol{B} \end{bmatrix}^{+}$

$$= \frac{1}{4\pi\varepsilon_0} {}^{-}\begin{bmatrix} \underline{\gamma \frac{\partial}{\partial \mathrm{c}t} - \gamma\beta_x \frac{\partial}{\partial x}} & \\ & -(\underline{\gamma \frac{\partial}{\partial x} - \gamma\beta_x \frac{\partial}{\partial \mathrm{c}t}}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \frac{q\gamma}{r} & \\ & -(\frac{q\gamma\beta_x}{r}, 0, 0) \end{bmatrix}^{+}.$$

Where $\frac{\partial}{\partial \mathrm{c}t}, \frac{\partial}{\partial x}, \cdots$ and r have the same meaning as in the case (i).

$$= \frac{1}{4\pi\varepsilon_0} {}^{-}\begin{bmatrix} (-\gamma\beta_x \frac{\partial}{\partial x})(\frac{q\gamma}{r}) & \\ & -(\gamma \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})(\frac{q\gamma}{r}) \end{bmatrix}^{+}$$

$$+ \frac{1}{4\pi\varepsilon_0} {}^{-}\begin{bmatrix} -(\gamma \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \bullet (-\frac{q\gamma\beta_x}{r}, 0, 0) & \\ & (-\gamma\beta_x \frac{\partial}{\partial x})(-\frac{q\gamma\beta_x}{r}, 0, 0) \\ & +\mathrm{i}(\gamma \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}) \times (-\frac{q\gamma\beta_x}{r}, 0, 0) \end{bmatrix}^{+}$$

$$= \frac{1}{4\pi\varepsilon_0} {}^{-}\begin{bmatrix} \frac{\gamma\beta_x q\gamma x}{r^3} & \\ & (\frac{\gamma q\gamma x}{r^3}, \frac{q\gamma y}{r^3}, \frac{q\gamma z}{r^3}) \end{bmatrix}^{+} + \frac{1}{4\pi\varepsilon_0} {}^{-}\begin{bmatrix} -\frac{\gamma q\gamma\beta_x x}{r^3} & \\ & (-\frac{\gamma\beta_x q\gamma\beta_x x}{r^3}, \mathrm{i}\frac{q\gamma\beta_x z}{r^3}, -\mathrm{i}\frac{q\gamma\beta_x y}{r^3}) \end{bmatrix}^{+}$$

$$= \frac{q}{4\pi\varepsilon_0 r^3} {}^{-}\begin{bmatrix} 0 & \\ & (x, \gamma y + \mathrm{i}\gamma\beta_x z, \gamma z - \mathrm{i}\gamma\beta_x y) \end{bmatrix}^{+}$$

$$= \frac{q}{4\pi\varepsilon_0 r^3} {}^{-}\begin{bmatrix} 0 & \\ & (x, \gamma y, \gamma z) - \mathrm{i}\gamma\beta_x (0, -\mathrm{i}z, \mathrm{i}y) \end{bmatrix}^{+}$$

(Cf. when the charge “en bloc” $\vec{q} = {}^{-}\begin{bmatrix} q\gamma & \\ & q\gamma\beta \end{bmatrix}^{-}$ is back building then the field is

$$ {}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-\mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+} = \frac{1}{4\pi\varepsilon_0}\, {}^{-}\begin{bmatrix} \frac{\partial}{\partial ct} & \\ & -(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}) \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \frac{\gamma q}{r} & \\ & -(\frac{\gamma\beta_x q}{r},0,0) \end{bmatrix}^{+}. \quad) $$

The force at the point $(x,y,0)$ is

$$ {}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = \frac{q}{4\pi\varepsilon_0 r^3}\, {}^{-}\begin{bmatrix} 0 & \\ & (x,\gamma y,-\mathrm{i}\gamma\beta_x y) \end{bmatrix}^{+} {}^{-}\begin{bmatrix} q'\gamma & \\ & (q'\gamma\beta_x,0,0) \end{bmatrix}^{-} $$

$$ = \frac{q}{4\pi\varepsilon_0 r^3}\, {}^{-}\begin{bmatrix} q'\gamma\beta_x x & \\ & (q'\gamma x, \gamma y\cdot q'\gamma-\gamma\beta_x y\cdot q'\gamma\beta_x, -\mathrm{i}\gamma\beta_x y\cdot q'\gamma+\mathrm{i}\gamma y\cdot q'\gamma\beta_x) \end{bmatrix}^{-} $$

$$ = \frac{qq'}{4\pi\varepsilon_0 r^3}\, {}^{-}\begin{bmatrix} \gamma\beta_x x & \\ & (\gamma x, y, 0) \end{bmatrix}^{-}. $$

Theorem 2.

The force $f_r = -\dfrac{\mu_0 j_s\cdot j_s'}{4\pi r^2}$ by the stream charge is the same type $F_r = -\dfrac{\mu_0 Idx\cdot I'dx'}{4\pi r^2}$ between two parallel currents I and I' in the conductor.

(Proof)

We take the current I (coulomb/sec) in two conductors with distance "r". There is moving charges density ρ_q (coulomb/m) in these conductors. When this speed is $\boldsymbol{v}$ (m/sec), the current is $I=\rho_q\boldsymbol{v}$ (coulomb/sec) and the stream charge is $\dfrac{j_s}{c}=\rho_q dx\gamma\beta$ (coulomb).

For simplicity, we assume that the charge $q=\rho_q dx$ and its speed are homogeneous.

By the above case (ii), Example 3.

The field generated by the moving charge is devided into two parts

$$ {}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-\mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+}_{(\vec{q})} = {}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-\mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+}_{(q\gamma)} + {}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-\mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+}_{(q\gamma\boldsymbol{\beta})} $$

(1) The part of the field for the relativistic charge $q\gamma(=\rho_q\gamma dx)$

$$ {}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E}-\mathrm{i}c\boldsymbol{B} \end{bmatrix}^{+}_{(q\gamma)} = \frac{q\gamma}{4\pi\varepsilon_0 r^3}\, {}^{-}\begin{bmatrix} \gamma\beta_x x & \\ & (\gamma x, y, z) \end{bmatrix}^{+} $$

$$E_t = \frac{q\gamma}{4\pi\varepsilon_0 r^3}\gamma\beta_x x\,,\ \boldsymbol{E} = \frac{q\gamma}{4\pi\varepsilon_0 r^3}(\gamma x, y, z) \ \text{ and }\ \mathrm{c}\boldsymbol{B} = \frac{q\gamma}{4\pi\varepsilon_0 r^3}(0,0,0)$$

Then the force to the relativistic charge $q'\gamma'$ at the point $(x, y, 0)$ is

$$^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-}_{(q\gamma,q'\gamma')} = \frac{qq'\gamma\gamma'}{4\pi\varepsilon_0 r^3}\ ^{-}\begin{bmatrix} \gamma\beta_x x & \\ & (\gamma x, y, 0) \end{bmatrix}^{+}.$$

Thus the force to the stream charge $q'\gamma\beta$ at the point $(x, y, 0)$ is

$$^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-}_{(q\gamma,q'\gamma\boldsymbol{\beta})} = \frac{qq'\gamma\gamma'\beta_x}{4\pi\varepsilon_0 r^3}\ ^{-}\begin{bmatrix} \gamma x & \\ & (\gamma\beta_x x, 0, \mathrm{i}y) \end{bmatrix}^{+}.$$

(2) The part of the field for the stream charge $q\gamma\beta(=\rho_q\gamma\beta dx = \frac{j_s}{\mathrm{c}})$

$$^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E} - \mathrm{ic}\boldsymbol{B} \end{bmatrix}^{+}_{(q\gamma\boldsymbol{\beta})} = \frac{q\gamma\beta_x}{4\pi\varepsilon_0 r^3}\ ^{-}\begin{bmatrix} -\gamma x & \\ & (-\gamma\beta_x x, \mathrm{i}z, -\mathrm{i}y) \end{bmatrix}^{+},\ \varepsilon_0\mu_0 = \frac{1}{\mathrm{c}^2}$$

$$= \frac{\mathrm{c}\mu_0 j_s}{4\pi r^3}\ ^{-}\begin{bmatrix} -\gamma x & \\ & (-\gamma\beta_x x, \underline{0,0}) - \mathrm{i}\underline{\underline{(0,-z,y)}} \end{bmatrix}^{+},\ \frac{1}{4\pi\varepsilon_0} = \frac{\mathrm{c}^2\mu_0}{4\pi}$$

$E_t = -\frac{\mathrm{c}\mu_0 j_s}{4\pi r^3}\gamma x\,,\ \boldsymbol{E} = \frac{\mathrm{c}\mu_0 j_s}{4\pi r^3}(-\gamma\beta_x, 0, 0)$ and $\mathrm{c}\boldsymbol{B} = \frac{\mathrm{c}\mu_0 j_s}{4\pi r^3}(0, -z, y)$,the latter is similar to a magnetic field which is generated by the current, that is,

$$\mathrm{c}\boldsymbol{B} = \frac{\mathrm{c}^2\mu_0}{4\pi r^3}\rho_q\gamma\beta dx(0,-z,y) = \frac{\mathrm{c}^2\mu_0}{4\pi r^3}\rho_q\gamma\beta(dx,0,0)\times(0,y,z) = \frac{\mathrm{c}\mu_0}{4\pi}\frac{j_s}{r^2}\times\frac{\boldsymbol{r}}{r}.$$

Then the force to the relativistic charge $q'\gamma'$ at the point $(x, y, 0)$ is

$$^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-}_{(q\gamma\boldsymbol{\beta},q'\gamma')} = \frac{qq'\gamma^2\beta_x}{4\pi\varepsilon_0 r^3}\ ^{-}\begin{bmatrix} -\gamma x & \\ & (-\gamma\beta_x x, 0, -\mathrm{i}y) \end{bmatrix}^{-}.$$

Thus the force to the stream charge $q'\gamma\beta$ at the point $(x, y, 0)$ is

$$^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-}_{(q\gamma,q'\gamma\boldsymbol{\beta})} = \frac{qq'\underline{\gamma^2\beta_x^{\,2}}}{4\pi\varepsilon_0 r^3}\ ^{-}\begin{bmatrix} -\gamma\beta_x x & \\ & (-\gamma x, -y, 0) \end{bmatrix}^{-} = \frac{\mu_0 j_s\cdot j_s'}{4\pi r^3}\ ^{-}\begin{bmatrix} -\gamma\beta_x x & \\ & (-\gamma x, -y, 0) \end{bmatrix}^{-}.$$

Therefore the force to the y-direction is $f_y = -\frac{\mu_0 j_s\cdot j_s'}{4\pi r^3}y\,,\ x = 0, y = r$.

Thus $f_y = -\frac{\mu_0 j_s \cdot j_s{}'}{4\pi r^2}$, $\underline{\gamma^2 \beta_x{}^2} = \sinh^2\Theta = 4\sinh^2\frac{\Theta}{2}\cosh^2\frac{\Theta}{2} = 4\sinh^2\frac{\Theta}{2} + \underline{\underline{4\sinh^4\frac{\Theta}{2}}}$

Q.E.D

Corollary.

We take account of the effect from the relativistic charge which generate the force of repulsion. Then the force is decreased by the amount $\underline{\underline{4\sinh^4\frac{\Theta}{2}}}$.

(Proof)

When the energy pours into the conductor and moves the charge q ,

then $\vec{q} = {}^{-}\begin{bmatrix} q & \\ & (0,0,0) \end{bmatrix}^{-}$ changes to ${}^{-}\begin{bmatrix} q\gamma & \\ & (q\gamma\beta_x, 0, 0) \end{bmatrix}^{-}$.

Therefore, the 4-dimensional charge in the conductor is

$$\Delta\vec{q} = {}^{-}\begin{bmatrix} q\gamma & \\ & (q\gamma\beta_x, 0, 0) \end{bmatrix}^{-} - {}^{-}\begin{bmatrix} q & \\ & (0,0,0) \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} q(\gamma-1) & \\ & (q\gamma\beta_x, 0, 0) \end{bmatrix}^{-}$$

Then its potential of the covariant vector is ${}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\boldsymbol{A} \end{bmatrix}^{+} = \frac{1}{4\pi\varepsilon_0}\,{}^{+}\begin{bmatrix} \frac{q(\gamma-1)}{r} & \\ & -(\frac{q\gamma\beta_x}{r}, 0, 0) \end{bmatrix}^{+}$.

Ths by the (1) and (2),Theorem 2, the field is ${}^{-}\begin{bmatrix} E_t & \\ & \boldsymbol{E} - i\mathrm{c}\boldsymbol{B} \end{bmatrix}^{+}$,

$$= \frac{q}{4\pi\varepsilon_0 r^3}\,{}^{-}\begin{bmatrix} 0 & \\ & (x, \gamma y + i\gamma\beta_x z, \gamma z - i\gamma\beta_x y) \end{bmatrix}^{+} - \frac{q}{4\pi\varepsilon_0 r^3}\,{}^{-}\begin{bmatrix} 0 & \\ & (x, y, z) \end{bmatrix}^{+}$$

$$= \frac{\mathrm{c}^2\mu_0 q}{4\pi r^3}\,{}^{-}\begin{bmatrix} 0 & \\ & (0, (\gamma-1)y + i\gamma\beta_x z, (\gamma-1)z - i\gamma\beta_x y) \end{bmatrix}^{+}$$

$$= \frac{\mathrm{c}^2\mu_0 q}{4\pi r^3}\,{}^{-}\begin{bmatrix} 0 & \\ & (0, \underline{(\gamma-1)y, (\gamma-1)z}) - i\underline{\underline{\gamma\beta_x (0, -z, y)}} \end{bmatrix}^{+} .$$

$E_t = 0$, $\boldsymbol{E} = \frac{\mathrm{c}^2\mu_0 q}{4\pi r^3}(0, (\gamma-1)y, (\gamma-1)z)$ and $\mathrm{c}\boldsymbol{B} = \frac{\mathrm{c}\mu_0 j_s}{4\pi r^3}(0, -z, y)$

Therefore the magnetic field of the stream charge in the straight conductors is

$$\mathrm{c}B_z = \frac{\mathrm{c}\mu_0 j_s}{4\pi}\int \frac{y\,dx}{(x^2+y^2)^{\frac{3}{2}}} = \frac{\mathrm{c}\mu_0 j_s}{2\pi r} , y = r .$$

Moreover, the force between two infinitesimal conductors with distance $y = r$ is

$$
{}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = \frac{\mathrm{c}\mu_0 q}{4\pi r^3}\,{}^{-}\begin{bmatrix} 0 & \\ & (0,(\gamma-1)y,-\mathrm{i}\gamma\beta_x y) \end{bmatrix}^{+}\,{}^{-}\begin{bmatrix} q'(\gamma-1) & \\ & (q'\gamma\beta_x,0,0) \end{bmatrix}^{-}
$$

$$
= \frac{\mathrm{c}\mu_0 qq'}{4\pi r^3}\,{}^{-}\begin{bmatrix} 0 & \\ & \begin{matrix} (0 \\ ,(\gamma-1)^2 y - \gamma\beta_x\gamma\beta_x y \\ ,-\mathrm{i}\gamma\beta_x(\gamma-1)\gamma\beta_x y + \mathrm{i}(\gamma-1)\gamma\beta_x y) \end{matrix} \end{bmatrix}^{-}
$$

$$
= \frac{\mathrm{c}\mu_0 qq'}{4\pi r^3}\,{}^{-}\begin{bmatrix} 0 & \\ & (0,-\underline{\underline{2(\gamma-1)}},0) \end{bmatrix}^{-},
$$

where $\underline{\underline{2(\gamma-1)}} = 2(\cosh\Theta - 1) = 4\sinh^2\dfrac{\Theta}{2}$ (Cf. Theorem 2).

(Q.E.D.)

Conclusion

The authors propose a new idea “en bloc” which is a component of the moving charge or particle in the Minkowski space by the Lorentz transformation.

This notation is very helpful for getting our ideas in shape. The following conclusions are drawn:

(1) We can describe the some event in any coordinate system on the Minkowski space.

(2) We clarify the relation between stream charge and current.

References

[1] Y. Takemoto, New Notation and Relativistic Form of the 4-dimensional Vector in Time-Space, Bull. of NBU Vol.34, No.1 (2006-Mar.) pp.32-38.

[2] Y. Takemoto and S. Shimamoto, The Positionality of the Electromagnetic and Gravitational Theory, Bull. of NBU Vol.40, No.1 (2012- Mar.) pp.1-11.