The rotation and the revolution of Mercury*

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Abstract

In the previous paper, authors showed that the resonance number of the planets in the solar system is 2-6 from the set of the Venus, the Earth, the Mars whose resonance number is 1. And the resonance number of Mercury is below 0.6, because the Mercury is very close to the Sun.

In this paper, the Mercury's behavior in the strong gravitation as the ratio of the revolution and the rotation is 2:3 and we find the point in which the speeds of the rotation and the revolution are same values.

1. Preliminaries.

(i)The equations of motion of Newton's type.

We consider the two-body problem concerned with the Sun and the planet.

We use the polar coordinate (t, r, θ, φ).

$$\begin{cases} t = t \\ x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

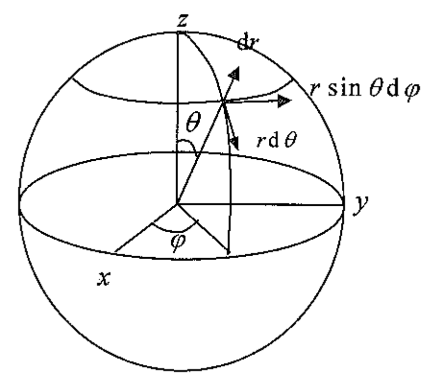

Fig. 1 The spherical coordinate.

Then the planet moves on the equator of the Sun. Therefore, we put, where Ω is a parameter relating to the angle of rotation on the orbit and $M_G = \dfrac{GM}{c^2}$ [2].

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The metric is $dc\tau^2 = dct^2 - dr^2 - (r\cosh\Omega d\phi)^2 + (rd\Omega)^2$ and the system of equations is

(1) $\frac{d^2ct}{d\tau^2} = -\frac{M_G}{r^2}\frac{dr}{d\tau}\frac{dct}{d\tau}$ $\cdots$ (the direction of time),

(2) $\frac{d^2r}{d\tau^2} = -\frac{M_G}{r^2}(\frac{dct}{d\tau})^2 + \frac{1}{r}\{(r\cosh\Omega\frac{d\phi}{d\tau})^2 - (r\frac{d\Omega}{d\tau})^2\}$ $\cdots$ (the direction of radius),

(3) $\frac{d}{d\tau}(r^2\frac{d\Omega}{d\tau}) = (\frac{M_G}{r^2}\frac{dct}{d\tau} - \sinh\Omega\frac{d\phi}{d\tau})(r^2\cosh\Omega\frac{d\phi}{d\tau})$ $\cdots$ (the longitude areal velocity),

(4) $\frac{d}{d\tau}(r^2\cosh\Omega\frac{d\phi}{d\tau}) = (\frac{M_G}{r^2}\frac{dct}{d\tau} - \sinh\Omega\frac{d\phi}{d\tau})(r^2\frac{d\Omega}{d\tau})$ $\cdots$ (the latitude areal velocity).

For having a good discussion, we translate the above equations of Newton's type to equations of Kepler's type.

(ii)The equations of motion of Kepler's type.

The system of equations is

(1)' $\frac{dct}{d\tau} = C_0 e^{\frac{M_G}{r}}$ $\cdots$ (the kinetic energy),

(2)' $\frac{d^2}{d\tau^2}(r\sinh\Omega) = -(\frac{M_G}{r^2}\frac{dct}{d\tau})(\tanh\Omega - r\cosh\Omega\frac{d\phi}{dct})\cosh\Omega(\frac{dct}{d\tau})$

$\cdots$ (the structure of space),

(3)' $(r^2\frac{d\Phi}{d\tau})^2 = (r^2\cosh\Omega\frac{d\phi}{d\tau})^2 - (r^2\frac{d\Omega}{d\tau})^2 = C^2$ $\cdots$ (the law of equal areas),

(4)' $r^2\cosh\Omega\frac{d\phi}{d\tau} = C\cosh\Theta'(\geqq 0)$, $r^2\frac{d\Omega}{d\tau} = -C\sinh\Theta'$

$\Theta' = \int(\sinh\Omega\frac{d\phi}{d\tau} - \frac{M_G}{r^2}\frac{dct}{d\tau})d\tau$ $\cdots$ (the internal rotation).

All information in physics is contained in this system of equations.

We put the angular velocity $\frac{d\Phi}{d\tau} = \sqrt{(\cosh\Omega\frac{d\phi}{d\tau})^2 - (\frac{d\Omega}{d\tau})^2}$, the orbit speed $r\frac{d\Phi}{d\tau}$ and the main equation $(\frac{d\frac{1}{r}}{d\Phi})^2 = (\frac{dr}{r^2 d\Phi})^2 = -\frac{c^2}{C^2} + \frac{C_0^2}{C^2}e^{2\frac{M_G}{r}} - \frac{1}{r^2}$.

We call C_0, C the energy constant and areas constant respectively.

2. The meaning of Θ' and Θ.

Minkowski metric is

$dc\tau^2 = dct^2 - dr^2 - (r\cosh\Omega d\phi)^2 + (rd\Omega)^2$.

Then

$$\frac{1}{(\frac{dt}{d\tau})^2} = 1 - (\frac{dr}{dct})^2 - (\frac{r\cosh\Omega d\phi}{dct})^2 + (\frac{rd\Omega}{dct})^2$$

$$\therefore \frac{dt}{d\tau} = \frac{1}{\sqrt{1-(\frac{dr}{dct})^2 - (\frac{r\cosh\Omega d\phi}{dct})^2 + (\frac{rd\Omega}{dct})^2}}. \quad \cdots (5)$$

On the other part, we put the speed of radial direction $\frac{v_r}{c} = \frac{dr}{dct}(=\tanh\Theta)$ and the speed of rotation direction $v_\phi = r\cosh\Omega\frac{d\phi}{dt}$, then Newton's composition is

$$\begin{pmatrix}\frac{v_r}{c}(=\tanh\Theta)\\ 0\\ 0\end{pmatrix} + \begin{pmatrix}0\\ \frac{v_\phi}{c}(=\tanh\Phi)\\ 0\end{pmatrix} = \begin{pmatrix}\frac{v_r}{c}(=\tanh\Theta)\\ \frac{v_\phi}{c}(=\tanh\Phi)\\ 0\end{pmatrix}.$$

When we use the relatively form, $\cosh\Theta = \frac{1}{\sqrt{1-(\frac{v_r}{c})^2}}$, $\cosh\Phi = \frac{1}{\sqrt{1-(\frac{v_\phi}{c})^2}}$ and relativistic composition of two directions (new) is

$$^{-}\begin{bmatrix}\cosh\Theta & \\ & (\sinh\Theta,0,0)\end{bmatrix}^{+}\ ^{-}\begin{bmatrix}\cosh\Phi & \\ & (0,\sinh\Phi,0)\end{bmatrix}^{-}$$

$$= ^{-}\begin{bmatrix}\cosh\Theta\cosh\Phi & \\ & (\sinh\Theta\cosh\Phi, \cosh\Theta\sinh\Phi, -i\sinh\Theta\sinh\Phi)\end{bmatrix}^{-}$$

$$= \cosh\Theta\cosh\Phi\ ^{-}\begin{bmatrix}1 & \\ & (\tanh\Theta, \tanh\Phi, -i\tanh\Theta\tanh\Phi)\end{bmatrix}^{-}.$$

Therefore, the time component is

$$\cosh\Theta\cosh\Phi = \frac{1}{\sqrt{1-(\frac{v_r}{c})^2}\sqrt{1-(\frac{v_\phi}{c})^2}} \quad \cdots (6)$$

Cf. the composition of the same direction speed v_1 and v_2, then Newton's composition is

$$\begin{pmatrix} \frac{v_1}{c}(=\tanh\Theta_1) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{v_2}{c}(=\tanh\Theta_2) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{v_1}{c}+\frac{v_2}{c}(=\tanh\Theta_1+\tanh\Theta_2) \\ 0 \\ 0 \end{pmatrix}.$$

We use the relatively form $\cosh\Theta_1 = \dfrac{1}{\sqrt{1-(\frac{v_1}{c})^2}}$, $\cosh\Theta_2 = \dfrac{1}{\sqrt{1-(\frac{v_2}{c})^2}}$, then the relativistic composition of a same direction is

$$\begin{bmatrix} \cosh\Theta_1 & \\ & (\sinh\Theta_1, 0, 0) \end{bmatrix}^{+} \begin{bmatrix} \cosh\Theta_2 & \\ & (\sinh\Theta_2, 0, 0) \end{bmatrix}$$

$$= \begin{bmatrix} \cosh\Theta_1\cosh\Theta_2 - \sinh\Theta_1\sinh\Theta_2 & \\ & (\cosh\Theta_1\sinh\Theta_2 + \sinh\Theta_1\cosh\Theta_2, 0, 0) \end{bmatrix}$$

$$= \begin{bmatrix} \cosh(\Theta_1+\Theta_2) & \\ & (\sinh(\Theta_1+\Theta_2), 0, 0) \end{bmatrix} \quad \text{by the additional theorem}$$

$$= \cosh(\Theta_1+\Theta_2)\begin{bmatrix} 1 & \\ & (\tanh(\Theta_1+\Theta_2), 0, 0) \end{bmatrix}.$$

This is a relativistic composition rule of speed.

Theorem 1.

We get the factorization of the metric as follows:

$$\text{(i)}\ (\frac{dc\tau}{dct})^2 = 1-(\frac{dr}{dct})^2 - (r\cosh\Omega\frac{d\phi}{dct})^2 + (r\frac{d\Omega}{dct})^2$$

$$= \{1-(\frac{dr}{dct})^2\}\{1-(r\cosh\Omega\frac{d\phi}{dct})^2\}$$

$$\text{(ii)}\ (\frac{dc\tau}{dr})^2 = (\frac{dct}{dr})^2 - 1 - (r\cosh\Omega\frac{d\phi}{dr})^2 + (r\frac{d\Omega}{dr})^2$$

$$= \{(\frac{dct}{dr})^2 - 1\}\{1-(r\frac{d\Omega}{dr})^2\}$$

$$\text{(iii)}\ (\frac{dc\tau}{r\cosh\Omega d\phi})^2 = (\frac{dct}{r\cosh\Omega d\phi})^2 - (\frac{dr}{r\cosh\Omega d\phi})^2 - 1 + (\frac{rd\Omega}{r\cosh\Omega d\phi})^2$$

$$= \{(\frac{dct}{r\cosh\Omega d\phi})^2 - 1\}\{1-(\frac{dr}{dct})^2\}$$

$$\text{(iv)}\ (\frac{dc\tau}{rd\Omega})^2 = (\frac{dct}{rd\Omega})^2 - (\frac{dr}{rd\Omega})^2 - (\frac{r\cosh\Omega d\phi}{rd\Omega})^2 + 1$$

$$= \{1-(\frac{dr}{rd\Omega})^2\}\{1-(\frac{r\cosh\Omega d\phi}{rd\Omega})^2\}$$

And we get $\tanh\Theta' = \tanh\Theta$ therefore $\Theta' = \Theta$ when the moment of inertia is zero.

(Proof)

By the metric, the energy is $$\frac{dt}{d\tau} = \frac{1}{\sqrt{1-(\frac{dr}{dct})^2 - (r\cosh\Omega\frac{d\phi}{dct})^2 + \underline{\underline{(r\frac{d\Omega}{dct})^2}}}}. \quad \cdots (5)$$

And by the composition of speed, its time component is

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{1-(\frac{v_\phi}{c})^2}\sqrt{1-(\frac{v_r}{c})^2}} = \frac{1}{\sqrt{1-(\frac{v_r}{c})^2 - (\frac{v_\phi}{c})^2 + (\frac{v_r}{c})^2\cdot(\frac{v_\phi}{c})^2}}$$

$$= \frac{1}{\sqrt{1-(\frac{dr}{dct})^2 - (r\cosh\Omega\frac{d\phi}{dct})^2 + \underline{\underline{(\frac{dr}{dct})^2\cdot(r\cosh\Omega\frac{d\phi}{dct})^2}}}}. \quad \cdots (7)$$

We compare energy (5) with the time component (7).

Then we get $(r\frac{d\Omega}{dct})^2 = (\frac{dr}{dct})^2\cdot(r\cosh\Omega\frac{d\phi}{dct})^2$ in this case the moment of inertia is zero.

And we adopt an anticlockwise rotation is positive.

$$r\frac{d\Omega}{dct} = -\frac{dr}{dct}\cdot r\cosh\Omega\frac{d\phi}{dct} \quad \cdots (8)$$

Therefore the energy formula of the metric $$\frac{dt}{d\tau} = \frac{1}{\sqrt{1-(\frac{dr}{dct})^2 - (\frac{r\cosh\Omega d\phi}{dct})^2 + (\frac{rd\Omega}{dct})^2}}$$

is factorized as $$\frac{dt}{d\tau} = \frac{1}{\sqrt{\{1-(\frac{dr}{dct})^2\}\{1-(\frac{r\cosh\Omega d\phi}{dct})^2\}}}.$$

Moreover using the same method, we get (ii)-(iv).

Therefore $\tanh\Theta = \dfrac{\mathrm{d}r}{\mathrm{d}ct} = \dfrac{-r\dfrac{\mathrm{d}\Omega}{\mathrm{d}ct}}{r\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct}} = \dfrac{-\dfrac{\mathrm{d}\Omega}{\mathrm{d}t}}{\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}t}} = \dfrac{-\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau}}{\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}}$ on the world line.

And by the formula (4)', $\tanh\Theta' = \dfrac{-r^2\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau}}{r^2\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}} = \dfrac{-\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau}}{\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau}}$.

We get $\tanh\Theta' = \tanh\Theta \therefore \Theta' = \Theta$ when the moment of inertia is zero. Q.E.D.

3. The stable point of rotation and revolution.

In the orbit the value of the energy mcC_0 is a constant and $\mathrm{d}\tau = \dfrac{r^2}{C}\mathrm{d}\Phi$.

We use the law of equal areas

(3)' $(r^2\dfrac{\mathrm{d}\Phi}{\mathrm{d}\tau})^2 = (r^2\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau})^2 - (r^2\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau})^2 = C^2 \cdots$ (the law of equal areas),

Then $r^2\dfrac{\mathrm{d}\Phi}{\mathrm{d}\tau} = C$. $\cdot\cdot\cdot$ (9)

And $(r^2\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau})^2 - (r^2\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau})^2 = (r^2\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}t})^2(\dfrac{\mathrm{d}t}{\mathrm{d}\tau})^2(1 - \dfrac{(r\dfrac{\mathrm{d}\Omega}{\mathrm{d}\tau})^2}{(r\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}\tau})^2})$

$$= \frac{(r^2\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}t})^2}{\{1-(\dfrac{\mathrm{d}r}{\mathrm{d}ct})^2\}\{1-(r\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct})^2\}}\{1-(\frac{\mathrm{d}r}{\mathrm{d}ct})^2\} = \frac{(r^2\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}t})^2}{1-(r\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct})^2} \text{ by (i).}$$

Then $\dfrac{(r\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct})^2}{1-(r\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct})^2} = (\dfrac{C}{cr})^2 \quad \therefore (r\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct})^2 = \dfrac{(\dfrac{C}{rc})^2}{1+(\dfrac{C}{rc})^2}$. $\cdot\cdot\cdot$ (10)

And besides $\dfrac{\mathrm{d}\Omega}{\mathrm{d}ct} = -\dfrac{\mathrm{d}r}{\mathrm{d}ct}\cdot\cosh\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct} = -\dfrac{\mathrm{d}r}{\mathrm{d}ct}\cdot\dfrac{\dfrac{C}{r^2\mathrm{c}}}{\sqrt{1+(\dfrac{C}{r\mathrm{c}})^2}} = \dfrac{1}{\sqrt{1+(\dfrac{C}{r\mathrm{c}})^2}}\dfrac{\mathrm{d}}{\mathrm{d}ct}(\dfrac{C}{r\mathrm{c}})$.

Then $\dfrac{1}{\sqrt{1+\sinh^2\Omega}}\dfrac{\mathrm{d}\sinh\Omega}{\mathrm{d}ct} = \dfrac{1}{\cosh\Omega}\dfrac{\mathrm{d}\sinh\Omega}{\mathrm{d}ct} = \dfrac{\mathrm{d}\Omega}{\mathrm{d}ct} = \dfrac{1}{\sqrt{1+(\dfrac{C}{r\mathrm{c}})^2}}\dfrac{\mathrm{d}}{\mathrm{d}ct}(\dfrac{C}{r\mathrm{c}})$.

Therefore, when we get $\sinh\Omega = \dfrac{C}{r\mathrm{c}}$.

$\therefore r\sinh\Omega = \dfrac{C}{\mathrm{c}},\ \cosh\Omega = \sqrt{1+\sinh^2\Omega} = \sqrt{1+(\dfrac{C}{r\mathrm{c}})^2},\ r^2\cosh^2\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct} = \dfrac{C}{\mathrm{c}}$. $\cdot\cdot\cdot$ (11)

We apply the planet orbit $\dfrac{1}{r} = \dfrac{1}{r_0}(1+e\cos\Phi)$, and by the (2)' of Kepler type.

$$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}(r\sinh\Omega) = -(\frac{M_G}{r^2}\frac{\mathrm{d}ct}{\mathrm{d}\tau})(\tanh\Omega - r\cosh\Omega\frac{\mathrm{d}\phi}{\mathrm{d}ct})\cosh\Omega(\frac{\mathrm{d}ct}{\mathrm{d}\tau}),\ M_G = \frac{GM}{\mathrm{c}^2}.$$

$$= -\frac{M_G}{r^3}(\frac{\mathrm{d}ct}{\mathrm{d}\tau})^2(r\sinh\Omega - r^2\cosh^2\Omega\frac{\mathrm{d}\phi}{\mathrm{d}ct}) \cdot\cdot\cdot \text{ This is like Hooke's law.}$$

(a) The revolution of the planet is stable then $r\sinh\Omega \equiv r^2\cosh^2\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct} \equiv \dfrac{C}{\mathrm{c}}$ when the moment of inertia is zero, but actually there is an effect of the moment of inertia.

(b) If there is a gap in the angular momentum of rotation $mr\sinh\Omega$ and the angular momentum of revolution $mr^2\cosh^2\Omega\dfrac{\mathrm{d}\phi}{\mathrm{d}ct}(=\dfrac{mC}{\mathrm{c}})$, then the force $\dfrac{M_G m\mathrm{c}^2}{r^2}(=\dfrac{GMm}{r^2})$ works to cancel the gap with this equation.

In this situation we assume that the momentum of rotation is stable, i.e. $m\sinh\Omega_* = \dfrac{mC}{r_*\mathrm{c}}$.

Then we seek the point of the center r_* such that

$$\mathbf{P} = \int\mathbf{F}d\tau = -\int_0^{r_\pi}(\frac{M_G}{r^2})(\frac{\mathrm{d}ct}{\mathrm{d}\tau})^2(m\sinh\Omega_* - mr\cosh^2\Omega\frac{\mathrm{d}\phi}{\mathrm{d}ct})\mathrm{d}\tau$$

$$=-\int_0^{\pi}(\frac{M_G}{C^2})(\frac{\mathrm{d}ct}{\mathrm{d}\tau})^2(\frac{mC}{r_* c}-\frac{mC}{rc})\mathrm{d}\Phi=0 \quad \text{by (9).}$$

In this point we expect that the formula $\cosh\Omega_*\frac{\mathrm{d}\phi_*}{\mathrm{d}ct}(revolution)=\frac{\tanh\Omega_*}{r_*}(rotation)$ will hold.

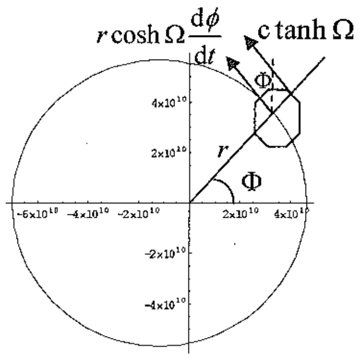

Fig. 2 The rotation and revolution.

Since the short axial direction is symmetrical, therefore we calculate the integral to the long axial direction in Φ.

Therefore we solve the equation $-\int_0^{\pi}(\frac{M_G}{C^2})(\frac{\mathrm{d}ct}{\mathrm{d}\tau})^2(\frac{mC\sin\Phi_*}{r_* c}-\frac{mC\sin\Phi}{rc})\mathrm{d}\Phi=0$.

Then $\int_0^{\pi}(\frac{\sin\Phi_*}{r_*}-\frac{\sin\Phi}{r})\mathrm{d}\Phi=0 \quad \therefore\frac{\pi\sin\Phi_*}{r_*}=\int_0^{\pi}\frac{1}{r}\sin\Phi\mathrm{d}\Phi$.

We use the elliptic orbit formula $\frac{1}{r}=\frac{1}{r_0}(1+e\cos\Phi)$ of the planet,

Then right side is

$$\int_0^{\pi}\frac{1}{r}\sin\Phi\mathrm{d}\Phi=\frac{1}{r_0}\int_0^{\pi}(1+e\cos\Phi)\sin\Phi\mathrm{d}\Phi=\frac{1}{r_0}\int_0^{\pi}(\sin\Phi+\frac{1}{2}e\sin 2\Phi)\mathrm{d}\Phi$$

$$=\frac{1}{r_0}\left[-\cos\Phi-\frac{1}{4}e\cos 2\Phi\right]_0^{\pi}=\frac{2}{r_0}.$$

Therefore $\frac{\pi\sin\Phi_*}{r_*}=\frac{2}{r_0} \quad \therefore\sin\Phi_*=\frac{2}{\pi}\frac{r_*}{r_0}$. · · · (12)

By the way, since $\frac{1}{r_*}=\frac{1}{r_0}(1+e\cos\Phi_*)$, $\cos\Phi_*=\frac{1}{e}(\frac{r_0}{r_*}-1)$. · · · (13)

Therefore we get the following equation for $\frac{r_*}{r_0}$ by the formula (12) and (13).

$$\{\frac{1}{e}(\frac{r_0}{r_*}-1)\}^2+(\frac{2}{\pi}\frac{r_*}{r_0})^2=1. \quad \cdots (14)$$

4. Examples

Example 1. The calculation using only eccentricity ($e=0.2056$) in Mercury.

The eccentricity is $e=0.2056$, then $\frac{r_*}{r_0}=0.852759$.

Then we get the center point, i.e. in this point, the rotation and evolution speeds coincide is

$$r_*=4.729589316763557\times10^{10}\ \mathrm{m} \quad \text{by} \quad r_0=5.546221882829519\times10^{10}\ \mathrm{m}.$$

In this point, the rotation angular speed and the revolution angular speed $\cosh^2\Omega_*\frac{\mathrm{d}\phi_*}{\mathrm{d}t}$ is $\frac{C}{r_*^2}=1.2128\times10^{-6}\ \mathrm{rad}\cdot\mathrm{sec}^{-1}$ by the area velocity $C=2.7129\times10^{15}\ \mathrm{m}^2\cdot\mathrm{sec}^{-1}$.

And the angle from perihelion is $\Phi_*=0.573866$ rad by the $\frac{r_0}{r_*}=1+e\cos\Phi_*$.

This means that by the area from perihelion is

$S=\frac{1}{2}\int_0^{\Phi_*}r^2\mathrm{d}\Phi=6.187177923346939\times10^{20}\ \mathrm{m}^2$, it takes $\frac{2S}{C}/24/3600=5.27924$ days after perihelion.

And the cycle of the rotation is $T_\omega(=\frac{2\pi}{\omega})=T=\frac{2\pi}{\frac{C}{r_*^2}}=\frac{2\pi r_*^2}{C}=59.9454$ days, but by measurement, $T_\omega=58.6462$ days because this result is not considered the "some" stable condition of the energy.

Example 2. The calculation using the condition ($e=0.2056$) of Example 1 and the additional condition of the ratio ($n:m=2:3$) of the revolution and the rotation.

The ratio of the revolution and the rotation is $n:m=2:3$ and the cycle of the revolution is $T=\frac{\pi r_s R_l}{\frac{C}{2}}=\frac{2\pi r_s R_l}{C}$ where $R_c=\frac{r_0}{1-e^2}$ is a length of the long radius and $r_s=\frac{r_0}{\sqrt{1-e^2}}$ is a length of the short radius and the cycle of the rotation is $T_\omega=\frac{2\pi}{\omega}=\frac{2\pi}{\frac{C}{r_*^2}}=\frac{2\pi r_*^2}{C}$.

Then $\frac{m}{n}=\frac{T}{T_{\omega}}=\frac{3}{2}$ $\therefore 3\frac{2\pi r_*^2}{C}=2\frac{2\pi r_s R_c}{C}$ $\therefore 3r_*^2=2r_s R_c=\frac{2r_0^2}{\sqrt{(1-e^2)^3}}$.

Therefore $(\frac{r_*}{r_0})^2=\frac{2}{3\sqrt{(1-e^2)^3}}$. $\cdots$ (15)

$$\therefore \frac{r_*}{r_0}=\sqrt{\frac{2}{3\sqrt{(1-e^2)^3}}}=0.843378.$$

By r_0 =5.546221882829519×10^10 m, $r_*=4.67756\times10^{10}$ m and

$\frac{C}{r_*^2}=1.23992\times10^{-6}$ rad·sec^{-1} by the area velocity $C=2.7129\times10^{15}$ m^2·sec^{-1}.

And the angle from perihelion is $\Phi_*=0.443523$ rad by the $\frac{r_0}{r_*}=1+e\cos\Phi_*$.

This means that by the area from perihelion is

$$S=\frac{1}{2}\int_0^{\Phi_*}r^2 d\Phi=4.7446407523117485\times10^{20}\ \mathrm{m}^2,$$

it takes $\frac{2S}{C}/24/3600=4.04844$ days after perihelion.

Example 3. The calculation using only the harmonic ratio $\frac{n}{m}=\frac{2}{3}$ which is the energy "level" of the revolution and the rotation of the Example 2 without the effects of the Venus beyond. In this case the eccentricity value e is not the usual one in the new orbit of the Mercury.

In the point r_* that the speeds of revolution and rotation are equal ($r_*\cosh\Omega_*\frac{d\phi_*}{dt}=c\tanh\Omega_*$) fills the equation (14) in the orbit $\frac{1}{r}=\frac{1}{r_0}(1+e\cos\Phi)$.

And from the ratio of the revolution and the rotation $\frac{2}{3}$, the same point r_* fills the equation (15).

Therefore from equations (14), (15), we get $\frac{(\frac{r_0}{r_*}-1)^2}{1-(\frac{2}{\pi}\frac{r_*}{r_0})^2}=e^2=1-(\frac{2}{3}(\frac{r_0}{r_*})^2)^{\frac{2}{3}}$

$$\therefore \frac{r_*}{r_0}=0.8462064 \quad \therefore e=0.215723.$$

This means that when the influence of external force does not come, the eccentricity of the Mercury orbit is $e=0.215723$.

5. Conclusion

In the Mercury, the eccentricity $e=0.2056$ has a close relation to the gravity force.

And the harmonic ratio $\frac{n}{m}=\frac{2}{3}$ is the parameter when getting a local minimal value like the energy.

Authors think this event may not be limited to only Mercury.

References

[1] Y. Takemoto, A Gauge Theory on the Anti-de Sitter Space, Bull. of NBU, Vol. 34, No.1 (1993-Feb.) pp. 99-115.
[2] Y. Takemoto, New Notation and Relativistic Form of the 4-dimensional Vector in Time-Space, Bull. of NBU, Vol. 34, No.1 (2006-Mar.) pp. 32-38.
[3] Y. Takemoto, A New Form of Equation of Motion for a Moving Charge and the Lagrangian, Bull. of NBU, Vol. 35, No.1 (2007-Mar.) pp. 1-9.
[4] Y. Takemoto, S. Shimamoto, The Gravitational Force And The Electromagnetic Force, Bull. of NBU, Vol. 43, No.1 (2015- Mar.) pp. 1-12.