~~プランク定数の角運動量をもつ振動~~

# The Oscillation with Planck Constant

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**Abstract**

We have three questions. The first question is that where the energy of light wave for example which radiated from the atomic in orbit transfer. The second question is that how mount the amplitude of the light wave.

We give the Planck's hypotheses $\Delta E = h\nu$ the another meaning. And the third question that why the Bohr radius is determined this value.

## 1. Preliminaries

**(i) A harmonic oscillator system is** $F = -kx$ **,** $F = m\alpha = m\dfrac{d^2x}{dt^2}$ **by the Newton's second law where** $k$ **is a positive constant.**

Solving this differential equation, we get the $x(t) = A\cos(\omega t + \phi)$,

$\omega(= 2\pi\nu) = \sqrt{\dfrac{k}{m}}\ , T = \dfrac{1}{\nu_{[s]}}$ . Because $x = a\cos\omega t = a\cos\sqrt{\dfrac{k}{m}}t$ ,

$(v =)x' = -a\omega\sin\omega t = -a\sqrt{\dfrac{k}{m}}\sin\sqrt{\dfrac{k}{m}}t$ , $\omega = 2\pi\nu = \sqrt{\dfrac{k}{m}}$ .

Then the energy W of system is $W = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2 = \dfrac{1}{2}ka^2\cos^2\omega t + \dfrac{1}{2}ma^2\omega^2\sin^2\omega t$ ,

$W = \dfrac{1}{2}ka^2 = \dfrac{1}{2}mv_0{}^2$ where $a$ is a max displacement and $v_0$ is a max speed.

Therefore we get $a = \sqrt{\dfrac{2W}{k}}$ and $v_0 = 2\pi a\nu = \sqrt{\dfrac{2W}{m}}$

$$mv^2 = ma^2\omega^2\sin^2\omega t = ma^2\omega^2(1 - \cos^2\omega t) = a^2k - a^2k\left(\frac{x}{a}\right)^2 = k(a^2 - x^2)$$

**(ii) The energy function of the "relativistic" harmonic oscillator system**

We put the energy function $E = \dfrac{m\mathrm{c}^2}{\sqrt{1-(\frac{v}{\mathrm{c}})^2}} e^{-\frac{k(a^2-x^2)}{2m\mathrm{c}^2}}$ (Constant), and when position is max i.e., $x = a$ and $v = 0$. Therefore at the general position,

$$m\mathrm{c}^2 (= \frac{m\mathrm{c}^2}{\sqrt{1-(\frac{0}{\mathrm{c}})^2}} e^{-\frac{k(a^2-a^2)}{2m\mathrm{c}^2}}) = \frac{m\mathrm{c}^2}{\sqrt{1-(\frac{v}{\mathrm{c}})^2}} e^{-\frac{k(a^2-x^2)}{2m\mathrm{c}^2}} = m\mathrm{c}^2(1+\frac{1}{2}(\frac{v}{\mathrm{c}})^2+\cdots)(1-\frac{k(a^2-x^2)}{2m\mathrm{c}^2}+\cdots)$$

$$= m\mathrm{c}^2(1+\frac{1}{2}(\frac{v}{\mathrm{c}})^2 - \frac{k(a^2-x^2)}{2m\mathrm{c}^2}+\cdots) = m\mathrm{c}^2 + \frac{1}{2}mv^2 - \frac{k}{2}(a^2-x^2)+\cdots$$

$$\therefore \frac{1}{2}mv^2 \doteqdot \frac{k}{2}(a^2-x^2)$$

**2. Some tools.**

**(i) Planck's hypotheses**

$\Delta E_{[kgm^2/s^2]} = h_{[kgm^2/s]} \nu_{[/s]}$ ∵ $h_{[kgm^2/s]} = 6.62607 \times 10^{-34}$ energy should be proportional to the frequency ν

**(ii) The surrounding frequency energy**

The electronic movement is mostly determined by the proton electric charge of a central nucleus.

Then in the circle orbit the 4-dimensional force $(f_t, f_x, f_y, f_z)$ on the electron is

$$\begin{bmatrix} f_t \\ \boldsymbol{f} \end{bmatrix}, \boldsymbol{f} = (f_x, f_y, f_z)$$

$$(= \frac{\mathrm{d}}{\mathrm{d}\mathrm{c}\tau} \begin{bmatrix} E \\ \mathrm{c}(p_x, p_y, p_z) \end{bmatrix}, P = (p_x, p_y, p_z))$$

$$= \begin{bmatrix} 0 \\ (E_x, 0, 0) \end{bmatrix} e \begin{bmatrix} \frac{u_0}{\mathrm{c}} \\ (0, \frac{u_y}{\mathrm{c}}, 0) \end{bmatrix}_{(x,0,0)},$$

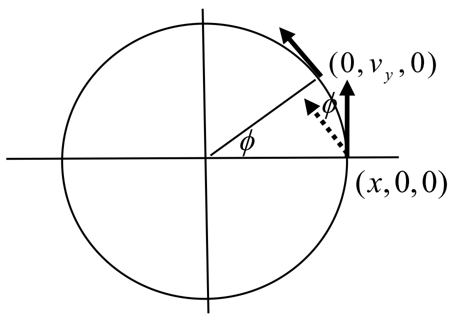


Fig. 1. Schematic diagram of the electric movement.

$$\frac{u_0}{\mathrm{c}} = \gamma = \frac{\mathrm{d}\mathrm{c}t}{\mathrm{d}\mathrm{c}\tau}, \quad \frac{u_y}{\mathrm{c}} = \gamma\beta_y = \frac{\mathrm{d}t}{\mathrm{d}\tau}\frac{\mathrm{d}y}{\mathrm{d}\mathrm{c}t} = \frac{\mathrm{d}y}{\mathrm{d}\mathrm{c}\tau}$$

$$= e \begin{bmatrix} 0 \\ (E_x\gamma, 0, -\mathrm{i}E_x\gamma\beta) \end{bmatrix}, \quad -eE_x = \frac{keQ}{r^2} (= \frac{m_e {v_y}^2}{r} = m_e v_y \cdot \frac{v_y}{r}).$$

Therefore the 4-dimensional force $(f_t, f_x, f_y, f_z)$ on the electron is $f_t = e \cdot 0 = 0$

.

$$=-\begin{bmatrix}0 & \\ & (p_x,0,\mathrm{i}p_z)\end{bmatrix}\frac{\mathrm{d}\varphi}{\mathrm{d}t},\ \frac{\mathrm{d}\varphi}{\mathrm{d}t}=\frac{v_y}{r},\ p_x=\frac{m_e v_y}{\sqrt{1-(\frac{v_y}{\mathrm{c}})^2}}=\frac{2\pi m_e r_1 v_y}{2\pi r_1\sqrt{1-(\frac{v_y}{\mathrm{c}})^2}}=\frac{h}{\lambda},\lambda=2\pi r_1,$$

$$p_z=\frac{m_e v_y}{\sqrt{1-(\frac{v_y}{\mathrm{c}})^2}}\frac{v_y}{\mathrm{c}},\quad E'(the\,surrounding\,energy)=h\nu=\frac{hv_y}{\lambda}=\frac{m_e\mathrm{c}^2(\frac{v_y}{\mathrm{c}})^2}{\sqrt{1-(\frac{v_y}{\mathrm{c}})^2}}=\mathrm{c}p_z$$

where $\lambda$ is the circumferential length, $v(=v_y)$ is the surrounding speed , $\nu$ is the surrounding frequency, $\lambda\nu=v$ . And at the Bohr radius $r(=r_1)$,

$$\nu=\frac{E'}{h}\fallingdotseq\frac{m_e\mathrm{c}^2(\frac{v_1}{\mathrm{c}})^2}{2\pi m r_1 v_1}=\frac{v_1(Speed)}{2\pi r_1(Length\ of\ circle)}=\frac{\frac{1}{2}r_1v_1(Speed\ of\ area)}{\pi r_1^2(Area\ of\ circle)}.$$

**(iii) Bohr radius**

We put $r_1$ the Bohr radius. By the balance in the orbit, $r_1 v_1^2=\dfrac{k_0e^2}{m_e}$ (1)

We put $R_0=\dfrac{k_0e^2}{m_e\mathrm{c}^2}=r_1(\dfrac{v_1}{\mathrm{c}})^2$ then $\dfrac{R_0}{r_1}=(\dfrac{v_1}{\mathrm{c}})^2=0.0000532513$ is a fine structure.

And by the Planck constant $h=2m_e\pi r_1 v_1$ (2)

Therefore $r_1=\dfrac{h^2}{(2\pi)^2 m_e k_0 e^2}=5.2923\times10^{-11}$

**(iv) The meanings of $R_0=\dfrac{k_0e^2}{m_e\mathrm{c}^2}$**

**a. The relativistic angular moment.**

By the balance the in the n-orbit, $r_n v_n^2=\dfrac{k_0e^2}{m_e}$ (3)

And by the "relativistic" Planck constant $nh=2m_e\pi r_n\dfrac{m_e\mathrm{c}\frac{v_n}{\mathrm{c}}}{\sqrt{1-(\frac{v_n}{\mathrm{c}})^2}}$

$$(2\pi r_n\mathrm{c}m_e v_n)^2=(nh)^2(\mathrm{c}^2-v_n^2)\qquad(4)$$

By (1) and (2). Then $(2\pi r_n\mathrm{c}m_e)^2\dfrac{k_0e^2}{m_e r_n}=(nh)^2(\mathrm{c}^2-\dfrac{k_0e^2}{m_e r_n})$

$$k_0e^2(2\pi\mathrm{c}m_e)^2r_n^2-(n\mathrm{c}h)^2m_e r_n+k_0e^2(nh)^2=0.$$

Therefore we get the orbital radius of electron

$$r_n=\frac{(n\mathrm{c}h)^2m_e\pm\sqrt{\{(n\mathrm{c}h)^2m_e\}^2-4k_0e^2(2\pi\mathrm{c}m_e)^2k_0e^2(n\mathrm{h})^2}}{2k_0e^2(2\pi\mathrm{c}m_e)^2}$$

$$= \frac{(nh)^2\left(1+\sqrt{1-\left(\frac{4\pi k_0 e^2}{n\mathrm{hc}}\right)^2}\right)}{2(2\pi)^2 k_0 e^2 m_e} \quad \text{(Cf. } r_n = \frac{(nh)^2}{(2\pi)^2 k_0 e^2 m_e} \text{ in classical).}$$

Especially $n=1$, we get $r_1 = \dfrac{h^2\left(1+\sqrt{1-\left(\frac{4\pi k_0 e^2}{\mathrm{hc}}\right)^2}\right)}{2(2\pi)^2 k_0 e^2 m_e} = 5.29137856\times 10^{-11}$

Moreover the formula of $r_n$ is indicated the limited radius in a circle orbit.

That is to say, when the case $\left(\dfrac{4\pi k_0 e^2}{nh\mathrm{c}}\right)^2 = 1$, then $n = \dfrac{4\pi k_0 e^2}{h\mathrm{c}} = \dfrac{1}{68.5165}$, therefore

$$r_s = \frac{(nh)^2}{2(2\pi)^2 k_0 e^2 m_e} = \frac{2k_0 e^2}{m_e \mathrm{c}^2} (= 2R_0) = 5.636\times 10^{-15} \quad \text{(the limited radius).}$$

**b. The energy function.**

We differentiate the Energy function $F(r)(=m_e\mathrm{c}C_0) = \dfrac{m_e\mathrm{c}^2}{\sqrt{1-\frac{k_0e^2}{m_e\mathrm{c}^2 r}}} e^{-\frac{k_0 e^2}{m_e \mathrm{c}^2 r}}$. Then

$$F'(r) = \frac{1}{2}\frac{m\mathrm{c}^2}{\sqrt{\left(1-\frac{k_0 e^2}{m_e\mathrm{c}^2 r}\right)^3}} \frac{k_0 e^2}{m_e \mathrm{c}^2 r^2} e^{-\frac{k_0 e^2}{m_e \mathrm{c}^2 r}} \left(1-\frac{2k_0 e^2}{m_e \mathrm{c}^2 r}\right) = 0.$$

Therefore, the minimum point is $r = \dfrac{2k_0 e^2}{m_e \mathrm{c}^2} (= 2R_0)$.

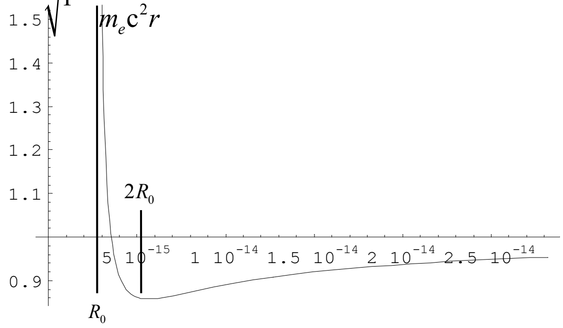


Fig. 2. The energy function.

**c. The limit of the speed circular moment.**

The "main equation" in the circle orbit is as follows;

(i) $\left(\dfrac{d\frac{1}{r}}{d\Phi}\right)^2 = \left(-\dfrac{\mathrm{c}^2}{C^2} + \dfrac{C_0{}^2}{C^2}\mathrm{e}^{\frac{2R_0}{r}} - \dfrac{1}{r^2}\right)\mathrm{e}^{\frac{2R_0}{r}} \equiv 0$, $\dfrac{1}{r^2} = -\dfrac{\mathrm{c}^2}{C^2} + \dfrac{C_0{}^2}{C^2}\mathrm{e}^{\frac{2R_0}{r}}$ and

(ii) $\dfrac{d^2\frac{1}{r}}{d\Phi^2} = \left(R_0\dfrac{C_0{}^2}{C^2}\mathrm{e}^{\frac{2R_0}{r}} - \dfrac{1}{r}\right)\mathrm{e}^{\frac{2R_0}{r}} + \left(-\dfrac{\mathrm{c}^2}{C^2} + \dfrac{C_0{}^2}{C^2}\mathrm{e}^{\frac{2R_0}{r}} - \dfrac{1}{r^2}\right)R_0\mathrm{e}^{\frac{2R_0}{r}} \equiv 0$, $\dfrac{1}{r} = R_0\dfrac{C_0{}^2}{C^2}\mathrm{e}^{\frac{2R_0}{r}}$

Therefore $\dfrac{1}{r^2} = -\dfrac{\mathrm{c}^2}{C^2} + \dfrac{1}{rR_0}$, $\dfrac{\mathrm{c}^2}{C^2}r^2 - \dfrac{1}{R_0}r + 1 = 0$ (5)

$$\frac{c^2}{\left(r\frac{\mathbf{v}}{\sqrt{1-(\frac{\mathbf{v}}{c})^2}}\right)^2}r^2-\frac{1}{R_0}r+1=0\,,\quad (\frac{\mathbf{v}}{c})^2=\frac{R_0}{r}\,. \tag{6}$$

This relation (6) is the same of the balance equation of the centrifugal force and the attractive force. Moreover, we solve the equation (5). Then,

$$r=\frac{\frac{1}{R_0}\pm\sqrt{\frac{1}{{R_0}^2}-4\frac{c^2}{C^2}}}{2\frac{c^2}{C^2}}\,,\quad 4\frac{c^2}{C^2}<\frac{1}{{R_0}^2}\quad \text{(from the discriminant).}$$

Therefore, the minimum of this radius and the maximum velocity are as follows;

When $\frac{c}{C}\left(=\frac{c}{r\frac{\mathbf{v}}{\sqrt{1-(\frac{\mathbf{v}}{c})^2}}}\right)=\frac{1}{2R_0}$,

then $r=\frac{\frac{1}{R_0}}{2\frac{c^2}{C^2}}=\frac{\frac{1}{R_0}}{2(\frac{1}{2R_0})^2}=2R_0$ and $\mathbf{v}=\frac{c}{\sqrt{2}}$.

**3. The energy of oscillation**

**(i) Planck constant (The angular moment of oscillation)**

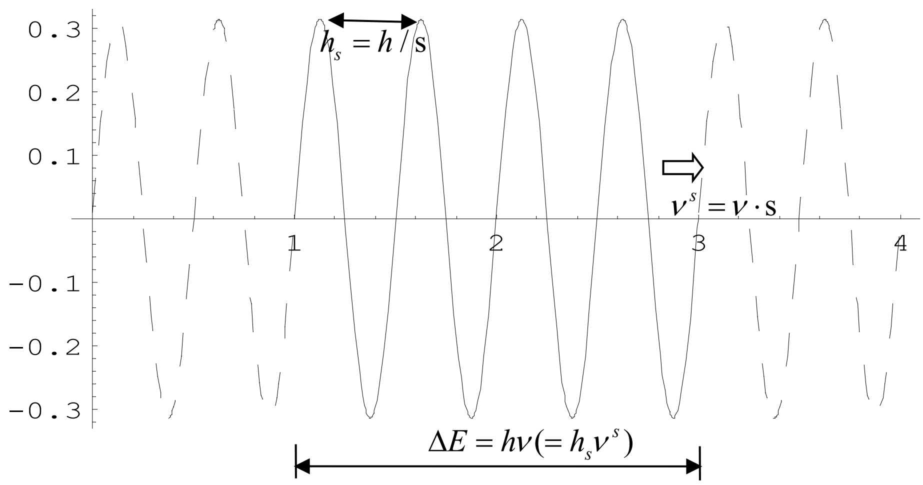


Fig. 3. The energy of light at the transfer (classic).

The Planck's hypotheses $\Delta E_{[kgm^2/s^2]} = h_{[kgm^2/s]} \nu_{[/s]}$ means that energy should be proportional to the frequency ν and the proportionality constant is $h_{[kgm^2/s]} = 6.62607 \times 10^{-34}$. **But we don't know the beginning and the end of the wave.**

$\nu$ is a frequent of the light. And we define $\nu^s$ only the number of the wave for one second.

Then we defined the energy $h_s = \dfrac{\Delta E}{\nu^s}{}_{[kgm^2/s^2]}$ which is the angular momentum for one second.

By the Planck's hypotheses, for any frequency $\nu$ the energy of angular motion $h_s = \dfrac{\Delta E}{\nu^s}{}_{[kgm^2/s^2]}$ is the same value and minimum energy.

We considered the harmonic oscillator which frequent is $\nu$ and its energy is $\underline{\underline{h_{s[kgm^2/s^2]}}}$.

$\omega(= 2\pi\nu) = \sqrt{\dfrac{k}{m_e}}$, $k = m_e\omega^2 = m_e(2\pi\nu)^2$ and $f = kx$ is the restoring force.

Therefore $\dfrac{1}{2}m_e v^2 + \dfrac{1}{2}kx^2 = h_s$, $a = \sqrt{\dfrac{2h_s}{k}} = \sqrt{\dfrac{2h_s}{(2\pi\nu)^2 m_e}} = \dfrac{1}{2\pi\nu}\sqrt{\dfrac{2h_s}{m_e}} = \dfrac{1}{\omega}\sqrt{\dfrac{2h_s}{m_e}}$ is the max of displacement(or radius) and $v_0 = 2\pi a\nu = \sqrt{\dfrac{2h_s}{m_e}} = 0.0381416_{[m/s]}$ is the speed.

**(ii) Difference energy in the transfer orbit**

The orbit of electron transfer from the parabolic(or elliptic) orbit to the circle orbit with some opportunity.

$\dfrac{2}{r_{[m\to n]}} \doteqdot \dfrac{1}{r_n} + \dfrac{1}{r_m}$, At the same speed point $r_{[m\to n]}$.

When the orbit is the parabolic, $m = \infty$, $r_{[\infty\to n]} = 2r_n$

and the speed of electron at the perihelion point is $v_n$.

Then we get the frequency of the electron as follows;

$\bar{\nu}_n = \dfrac{v_n}{2\pi(2r_n)} = \dfrac{\frac{1}{2}(2r_n)v_n}{\pi(2r_n)^2} (= \dfrac{Speed\ of\ area}{Area\ of\ circle})$, $\bar{\nu}_1 = \nu_1$ and

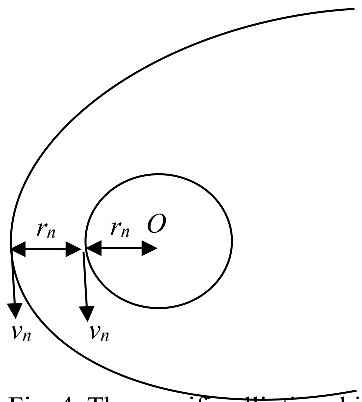

Fig. 4. The specific elliptic orbi

$$\Delta E_{n[kgm^2/s^2]} = m_e c^2 - \frac{m_e c^2}{\sqrt{1-(\frac{v_n}{c})^2}} e^{-\frac{R_0}{r_n}} \doteqdot \frac{1}{2} m_e v_n{}^2 = h_{s[kgm^2/s^2]}\, \nu_{n\,[-]}^{s} (= h_{[kgm^2/s]}\, \nu_{n\,[s^{-1}]})$$

We considered the harmonic oscillator which frequent is $\nu_n$ and its energy is $\underline{\underline{h_{s[kgm^2/s^2]}}}$.

Then the radius is $a_{n[m]} = \sqrt{\frac{2h_s}{k_{n[kg/s^2]}}} = \sqrt{\frac{2h_s}{(2\pi\nu_n)^2 m_e}} = \frac{1}{2\pi\nu_n}\sqrt{\frac{2h_s}{m_e}} = \frac{1}{\omega_n}\sqrt{\frac{2h_{s[kgm^2/s^2]}}{m_e}}$

and the speed is constant $v_{r_n,0} = a_n\omega_n = \sqrt{\frac{2h_s}{m_e}} (= v_0 = 0.0381416_{[m/s]})$ and same value.

By the relation $\Delta E_{[kgm^2/s^2]} \doteqdot \frac{1}{2} m_e v_n{}^2 = h_{s[kgm^2/s^2]} \nu_{n[-]}^s$, $\frac{1}{2} m_e v_1{}^2 = h_{s[kgm^2/s^2]} \nu_{1[-]}^s$

then $R_0 = r(\frac{v}{c})^2 = r_n \frac{2h_{s[kgm^2/s^2]}\nu_{n[-]}^s}{m_e c^2} = r_1 \frac{2h_{s[kgm^2/s^2]}\nu_{1[-]}^s}{m_e c^2}$. Therefore

$$\frac{a_n}{2r_n} = \underbrace{\frac{2h_s\nu_n^s}{2R_{0[m]}m_e c^2}}_{1/(2r_n)} \cdot \underbrace{\frac{1}{2\pi\nu_n}\sqrt{\frac{2h_s}{m_e}}}_{a_n} = \frac{\frac{m_e v_1{}^2}{\nu_1^s}\nu_n^s}{2r_1 v_1{}^2 m_e} \cdot \frac{1}{2\pi\nu_n}\sqrt{\frac{\frac{m_e v_1{}^2}{\nu_1^s}}{m_e}}$$

$$= \frac{\frac{1}{2}(2r_1)v_1}{\underline{\underline{\pi(2r_1)^2}}} \cdot \frac{\nu_n^s}{\nu_1^s \nu_n \sqrt{\nu_1^s}} = \underline{\underline{\nu_1}} \frac{\nu_n^s}{\nu_1^s \nu_n \sqrt{\nu_1^s}} = \frac{1}{\sqrt{\nu_1^s}} = 1.7434\times 10^{-8} \text{ (Constant)}$$

Therefore

$$\underline{\underline{\frac{a_n}{2r_n}}}\, \nu_{n[-]}^{s\ \frac{1}{2}} = \frac{1}{\sqrt{\nu_{1[-]}^s}} (\frac{\nu_{1[-]}^s}{n^2})^{\frac{1}{2}} = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots .$$

(The key map)

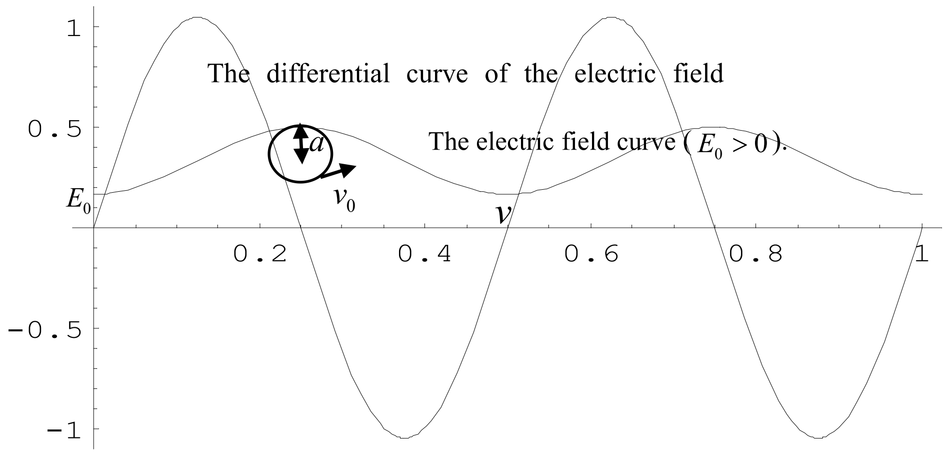


Fig. 5. The electric curve and its differential.

**(iii) For generally, "any" radius $r$ and $r(\frac{v}{\mathrm{c}})^2 = R_0$ (The balance in the circle).**

$$\Delta E_{[kgm^2/s^2]} = m_e \mathrm{c}^2 - \frac{m_e \mathrm{c}^2}{\sqrt{1-(\frac{v}{\mathrm{c}})^2}} e^{-\frac{R_0}{r}} \fallingdotseq \frac{1}{2} m_e v^2 = h_{s[kgm^2/s^2]} \nu^s{}_{[-]} (= h_{[kgm^2/s]} \nu_{[/s]})$$

We considered the harmonic oscillator which frequent is $\nu$ and its energy is $\underline{\underline{h_{s[kgm^2/s^2]}}}$.

Then the radius is $a = \sqrt{\frac{2h_s}{k}} = \sqrt{\frac{2h_s}{(2\pi\nu)^2 m_e}} = \frac{1}{2\pi\nu}\sqrt{\frac{2h_s}{m_e}} = \frac{1}{\omega}\sqrt{\frac{2h_s}{m_e}}$

and the speed is constant $v_{r,0} = a\omega = \sqrt{\frac{2h_s}{m_e}} (= v_0 = 0.0381416_{[m/s]}.)$

By the relation $\Delta E_{[kgm^2/s^2]} \fallingdotseq \frac{1}{2} m_e v^2 = h_s \nu^s, \frac{1}{2} m_e v_1{}^2 = h_s \nu_1^s$ then

$R_0 = r(\frac{v}{\mathrm{c}})^2 = r\frac{2h_s\nu^s}{m_e \mathrm{c}^2} = r_1 \frac{2h_s \nu_1^s}{m_e \mathrm{c}^2}$. Therefore, by the relation $\frac{1}{2} m_e v^2 = h_s \nu^s$, $R_0 = r(\frac{v}{\mathrm{c}})^2 = r\frac{2h_s\nu^s}{m_e \mathrm{c}^2}$

$$\frac{a}{2r} = \underbrace{\frac{2h_s\nu^s}{2R_0 m_e c^2}}_{1/(2r)} \cdot \underbrace{\frac{1}{2\pi\nu}\sqrt{\frac{2h_s}{m_e}}}_{a} (Constant)$$

$$= \frac{\frac{m_e v_1{}^2}{\nu_1^s}\nu^s}{2r_1 v_1{}^2 m_e} \cdot \frac{1}{2\pi\nu}\sqrt{\frac{\frac{m_e v_1{}^2}{\nu_1^s}}{m_e}} = \underline{\underline{\frac{\frac{1}{2}(2r_1)v_1}{\pi(2r_1)^2}}} \cdot \frac{\nu^s}{\nu_1^s \nu \sqrt{\nu_1^s}} = \underline{\underline{\nu_1}} \frac{\nu^s}{\nu_1^s \nu \sqrt{\nu_1^s}} = \frac{1}{\sqrt{\nu_1^s}}$$

Therefore we put $\underline{\underline{\frac{a}{2r}}}(\nu^s)^{\frac{1}{2}} = \frac{1}{\sqrt{\nu_1^s}}(\frac{\nu_1^s}{x^2})^{\frac{1}{2}} = \frac{1}{x}$.

then $\Delta E \fallingdotseq \frac{1}{2} m_e v^2 = h_s \nu^s = \frac{1}{2} m_e \cdot (2\pi a\nu)^2 \nu^s = \frac{1}{2} m_e \cdot \underbrace{(2\pi(2r)\nu)^2}_{The\ speed} \times \underbrace{(\frac{a}{2r}\sqrt{\nu^s{}_{[-]}})^2}_{The\ gap}$

$$(\frac{a}{2r})^2 \nu^s{}_{[-]} = (\frac{a}{2r}\sqrt{\nu^s{}_{[-]}})^2 = \frac{1}{x^2}$$

**Examples 1 (The resonance)**

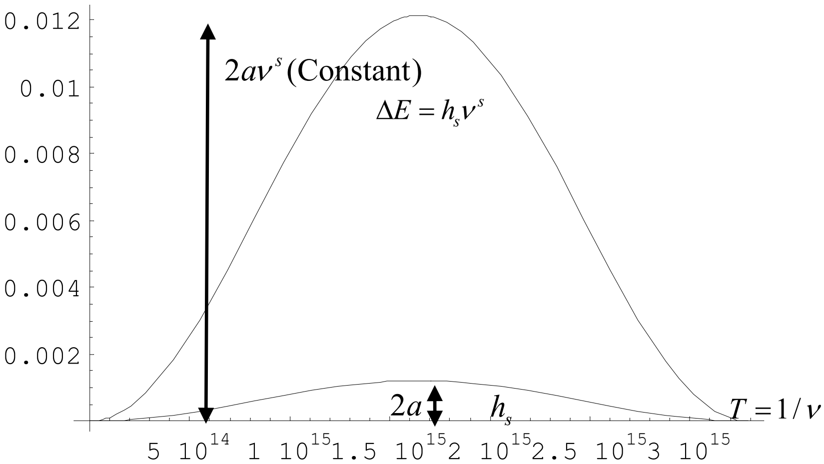


Fig. 6. The one wave (the light quantum).

We put the $(\frac{a}{2r})^2 \nu = \frac{1}{x^2}{}_{[/s]}$ , $a = \sqrt{\frac{2h_s}{k}} = \sqrt{\frac{2h_s}{(2\pi\nu)^2 m_e}} = \frac{1}{2\pi\nu}\sqrt{\frac{2h_s}{m_e}} = \frac{1}{\omega}\sqrt{\frac{2h_s}{m_e}}$

When $x = 1$, $\nu_1 = (\frac{2r}{a})^2{}_{[-]}(\frac{1}{1})^2{}_{[/s]} = 3.2898 \times 10^{15}{}_{[/s]}$

Then $a_1 = \frac{1}{2\pi\nu_1}\sqrt{\frac{2h_s}{m_e}} = 1.8452 \times 10^{-18}{}_{[m]}$ . Therefore,

$r_1 = \frac{1}{2}a_1\nu_1^{\frac{1}{2}} = \frac{1.8452 \times 10^{-18}}{2}\sqrt{3.2898 \times 10^{15}}{}_{[m]} = 5.2917 \times 10^{-11}{}_{[m]}$ (Bohr radius).

$$\Delta E_1 \doteqdot \frac{1}{2}m_e v_1^2 = \frac{1}{2}m_e v_1 (2\pi(2r_1)\nu_1 \cdot \frac{1}{1}) = \underbrace{2\pi m_e r_1 v_1}_{h} \cdot \nu_1$$

When $x = 2$, $\nu_2 = (\frac{2r}{a})^2(\frac{1}{2})^2{}_{[/s]} = 8.2246 \times 10^{14}{}_{[/s]} = 3.28984 \times 10^{15}{}_{[/s]} \times \frac{1}{4}$

$$\Delta E_2 \doteqdot \frac{1}{2}m_e v_2^2 = \frac{1}{2}m_e v_2 (2\pi(2r_2)\nu_2 \cdot \frac{1}{2}) = 2\pi m_e r_2 v_2 \cdot \frac{1}{2}\nu_2 = \underbrace{2\pi m_e r_1 v_1}_{h} \nu_2$$

When $x = n$, $\nu_n = (\frac{2r}{a})^2(\frac{1}{n})^2{}_{[/s]} = 3.28984 \times 10^{15}{}_{[/s]} \times \frac{1}{n^2}$, $a_n = \frac{1}{2\pi\nu_n}\sqrt{\frac{2h_s}{m_e}}$ , $r_n = \frac{1}{2}a_n\nu_n^{\frac{1}{2}}$

$$\Delta E_n \doteqdot \frac{1}{2}m_e v_n^2 = \frac{1}{2}m_e v_n (2\pi(2r_n)\nu_n \cdot \frac{1}{n}) = 2\pi m_e r_n v_n \cdot \frac{1}{n}\nu_n = \underbrace{2\pi m_e r_1 v_1}_{h} \nu_n$$

$\nu_n = \frac{\Delta E_n}{h}(= \frac{c}{\lambda_n})$ and in the moving of electron $v_n = 2\pi(2r_n)\nu_n \cdot \frac{1}{n} = \frac{v_1}{n}$ , $\lambda_{n,e} = \frac{v_n}{\nu_n}$

Therefore we get $h = 2\pi m r_1 v_1$ . and this is **the light quantum for "one action one wave"**.

**Examples 2 (The impact acceleration and the acceleration quantity)**

The orbit of electron transfer from the parabolic (or elliptic) orbit to the circle orbit with some opportunity.

$\frac{2}{r_{[m\to n]}} \doteqdot \frac{1}{r_n} + \frac{1}{r_m}$ At the same speed point $r_{[m\to n]}$ .

In the parabolic (or elliptic) orbit, the speed of electron is $v_n$ .

$$\nu_n = \frac{v_n}{2\pi(2r_n)} = \frac{\frac{1}{2}(2r_n)v_n}{\pi(2r_n)^2}$$

And we get the frequency of the electron at the perihelion point as follows;

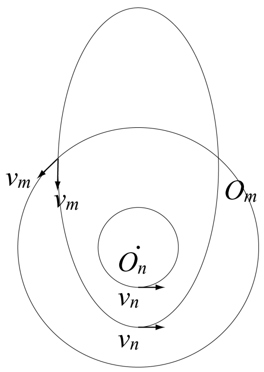


Fig. 7. The specific elliptic orbit.

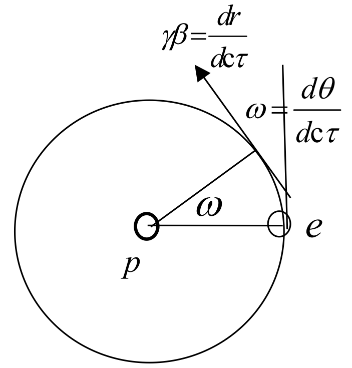


Fig. 8. The velocity and the acceleration on circle.

$$\underbrace{\frac{d}{d\tau}\,{}^{-}\left[\frac{dr}{d\tau}\right]^{-}}_{\text{the acceleration}} = \underbrace{{}^{-}\left[E\cdot\frac{e}{m_e\mathrm{c}}\right]^{+}}_{\text{the acceleration quantity}(\vec{E})_{[/s]}} \cdot \underbrace{{}^{-}\left[\frac{dr}{d\tau}\right]^{-}}_{\text{the speed}} \quad , \; {}^{-}\left[\frac{dr}{d\tau}\right]^{-}_{\text{(the first speed } v_n)} = {}^{-}\left[(\gamma\beta)_y\right]^{-}$$

$$\therefore \; \underline{\underline{{}^{-}\left[E\frac{e}{m_e\mathrm{c}}\right]^{+}}}_{[/s]} = \frac{d}{d\tau}\,{}^{-}\left[\frac{dr}{d\tau}\right]^{-}_{[m/s^2]} \cdot {}^{-}\left[\frac{dr}{d\tau}\right]^{-,-1}_{[s/m]} = {}^{-}\left[\frac{\mathrm{d}(\gamma\beta)_y}{(\gamma\beta)_y\,\mathrm{d}\tau}(=\omega_n = 2\pi\nu_n)\right]^{+}$$

$$(\nu_n \underset{Circle}{=} \frac{\mathrm{c}\cdot\tanh\Omega}{2\pi(2r_n)}) = \frac{v_n}{2\pi(2r_n)} = \frac{\frac{1}{2}(2r_n)v_n}{\pi(2r_n)^2}(= \frac{Speed\ of\ area}{Area\ of\ circle})$$

Therefore the acceleration quantity which is **impact acceleration** gives frequency $\nu_n$.

## 4. Conclusion

It is that Planck is saying implicitly in Planck's hypothesis as “Any frequency gets the energy of angular motion with the same minimum value (Planck's constant)”.

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