The Planet and The Electron*

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Abstract

In the previous paper, we reported that solve the orbit equation of an electron and find two type resonations i.e., (i) Bohr and (ii) a flower orbit resonations. In this paper, we study the planet in gravitation by solving the orbit equation which change the $R_0 = \frac{k_0 e^2}{mc^2}$ (in the electromagnetic) to the $M_G = \frac{GM}{c^2}$ (in the gravity). We calculated Mercury orbit and perihelion and bending of light by solving the orbit equation. This result is corresponding to the relativity theory.

1. Preliminaries

1.1. The similar to the Bohr radius

For the any planet m_p is a mass, v_p is a speed and r_p is a distance from the sun.

When the planet orbit is circle, by the balance in the orbit,

$$\frac{m_p {v_p}^2}{r_p}(\textit{the centrifubal force}) = \frac{GMm_p}{{r_p}^2}(\textit{the central force}) \left(= \frac{M_G m_p \mathrm{c}^2}{{r_p}^2} \right)$$
$$\therefore \; r_p {v_p}^2 = GM \qquad \therefore \; v_p = \sqrt{\frac{GM}{r_p}} = \sqrt{\frac{M_G \mathrm{c}^2}{r_p}} \; . \tag{1}$$

$$M_G = \frac{GM}{\mathrm{c}^2} = \frac{6.6743 \times 10^{-11} \times 1.9885 \times 10^{30}}{299792458^2} \fallingdotseq 1476.69197.$$

And by the planet moment (correspond to Planck constant in the electromagnetic),

$$h = 2\pi m_p r_p v_p = 2\pi m_p \sqrt{r_p GM} \; \left(= 2\pi m_p \mathrm{c} \sqrt{r_p M_G} \right). \tag{2}$$

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Therefore, the planet radius (correspond to Bohr radius in the electromagnetic) is

$$r_p=\frac{h^2}{(2\pi)^2 M_G (m_p\mathrm{c})^2}=\frac{1}{1476.69197}\left(\frac{h}{2\pi m_p \mathrm{c}}\right)^2,\quad \frac{\mathrm{c}^2}{GM}=\frac{1}{M_G}\fallingdotseq\frac{1}{1476.69197}\fallingdotseq 0.000677189\left[\frac{1}{m}\right].$$

1.2. The relativistic angular moment

In "the relativistic" case, we use the relativistic angular moment, $h_1=2\pi m_p r_p\dfrac{v_p}{\sqrt{1-\left(\dfrac{v_p}{\mathrm{c}}\right)^2}}$.

$$\hbar_1=m_p r_p\frac{v_p}{\sqrt{1-\left(\dfrac{v_p}{\mathrm{c}}\right)^2}}=m_p r_p u_p \text{ as the planet moment.}$$

When n-multiplication of $\hbar_1$. (i) $m_p r_n u_n^2=m_p r_n\left(\underline{\underline{\dfrac{v_n}{\sqrt{1-\left(\dfrac{v_n}{\mathrm{c}}\right)^2}}}}\right)^2 \underset{balance}{=} \dfrac{GM}{\mathrm{c}^2}m_p\left(\dfrac{\mathrm{d}ct}{\mathrm{d}\tau}\right)^2=\underline{\underline{\dfrac{M_G m_p \mathrm{c}^2}{\sqrt{1-\left(\dfrac{v_n}{\mathrm{c}}\right)^2}}}}$,

$r_n=n^2 r_p$, $v_n=\dfrac{v_p}{n}$, $u_n=\dfrac{v_n}{\sqrt{1-\left(\dfrac{v_n}{\mathrm{c}}\right)^2}}$. And (ii) $\hbar_n=m_p r_n\dfrac{v_n}{\sqrt{1-\left(\dfrac{v_n}{\mathrm{c}}\right)^2}}$ for $n=1,2,3,\cdots$.

Then (i)' $\left(\dfrac{v_n}{\mathrm{c}}\right)^2=\dfrac{M_G}{r_n}$ and (ii)' $\hbar_n=m_p \mathrm{c} r_n\dfrac{\dfrac{v_n}{\mathrm{c}}}{\sqrt{1-\left(\dfrac{v_n}{\mathrm{c}}\right)^2}}$.

$$\text{Therefore } (m_p \mathrm{c} r_n)^2\frac{M_G}{r_n}=\hbar_n^2\left(1-\frac{M_G}{r_n}\right). \tag{3}$$

$$\therefore (m_p\mathrm{c})^2 M_G r_n^2-\hbar_n^2 r_n+\hbar_n^2 M_G=0,\quad \hbar_n=\frac{h_n}{2\pi}. \tag{4}$$

We solve the equation (4), and then we get the orbital radius.

$$r_n=\frac{\hbar_n^2\pm\sqrt{(\hbar_n^2)^2-4(m_p\mathrm{c})^2 M_G \hbar_n^2 M_G}}{2(m_p\mathrm{c})^2 M_G}=\frac{\hbar_n^2\left(1+\sqrt{1-\left(\dfrac{2M_G m_p \mathrm{c}}{\hbar_n}\right)^2}\right)}{2(m_p\mathrm{c})^2 M_G}. \tag{5}$$

Especially $n=1$, we get

$$r_1(=r_p)=\frac{\hbar_1^2\left(1+\sqrt{1-\left(\dfrac{2M_G m_p \mathrm{c}}{\hbar_1}\right)^2}\right)}{2(m_p\mathrm{c})^2 M_G}\left(\fallingdotseq\frac{\left(\dfrac{\hbar_1}{m_p\mathrm{c}}\right)^2}{M_G}-M_G\fallingdotseq\frac{r_p^2 v_p^2}{GM}-M_G\right),\quad 2M_G\mathrm{c}=8.854022308\times10^{11}.$$

Moreover the formula (5) of r_n valid for $n<1$ and indicated the limited radius in a circle orbit.

That is to say, when the case $\left(\dfrac{2M_G m_p \mathrm{c}}{\hbar_n}\right)^2=1$, then

$$r_{n_s}=\frac{(2M_G m_p\mathrm{c})^2}{2(m_p\mathrm{c})^2 M_G}=2M_G\fallingdotseq 2953.38394\ (M_G\fallingdotseq 1476.69197),$$

$\dfrac{M_G}{r_p}\left(=\left(\dfrac{GM}{\hbar_p\mathrm{c}}\right)^2\right)=\left(\dfrac{v_p}{\mathrm{c}}\right)^2$ is "the fine structure?". And this point is minimal point of the E-function

$$E_0(r)\left(=\frac{C_0}{\mathrm{c}}\right)=\frac{1}{\sqrt{1-\dfrac{M_G}{r}}}e^{-\frac{M_G}{r}}[-].$$

And, Fig. 1 shows the E-function.

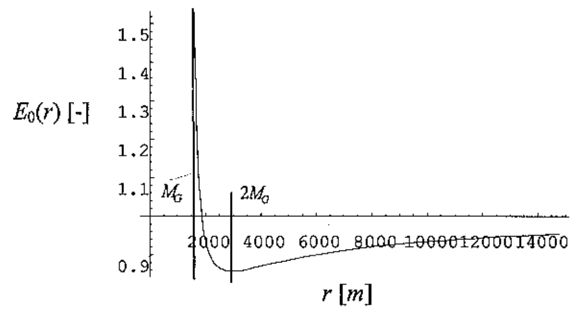

Fig. 1. The E-function.

And, by $\hbar_{n_s}=m_p n_s^2 r_p\dfrac{\dfrac{v_p}{n_s}}{\sqrt{1-\left(\dfrac{\frac{v_p}{n_s}}{\mathrm{c}}\right)^2}}=2M_G m_p\mathrm{c}$

$$\therefore (n_s r_p v_p)^2=(2M_G)^2\left(\mathrm{c}^2-\left(\frac{v_p}{n_s}\right)^2\right)$$

$$\therefore n_s^4 r_p^2 v_p^2-n_s^2(2M_G)^2\mathrm{c}^2+(2M_G v_p)^2=0$$

$$\therefore n_s^2=\frac{2M_G^2\mathrm{c}^2\pm\sqrt{(2M_G^2\mathrm{c}^2)^2-r_p^2 v_p^2(2M_G v_p)^2}}{r_p^2 v_p^2}$$

$$=\frac{2(M_G\mathrm{c})^2\left(1\pm\sqrt{1-\dfrac{r_p^2 v_p^4}{(M_G\mathrm{c}^2)^2}}\right)}{r_p^2 v_p^2}=\frac{2(M_G\mathrm{c})^2}{r_p^2 v_p^2}\text{ by (i)' }\left(\frac{v_n}{\mathrm{c}}\right)^2=\frac{M_G}{r_n}.$$

$$\therefore n_s r_p v_p=\sqrt{2}M_G\mathrm{c}\ \text{ and }\ r_p v_p^2=r_n v_n^2=M_G\mathrm{c}^2.$$

In this point, the speed is $v_{n_s} = \sqrt{\dfrac{\mathrm{GM}}{r_{n_s}}} = \sqrt{\dfrac{M_G c^2}{r_{n_s}}} = \sqrt{\dfrac{c^2}{2}} = \dfrac{c}{\sqrt{2}}$

for any planets (from Mercury to Pluto).

1.3. The limit of the speed in circular moment

The "orbit equation" in the circle orbit is as follows[4,5]:

$$\text{(i)}\quad \left(\frac{d\frac{1}{r}}{d\Phi}\right)^2 = -\frac{c^2}{C^2} + \frac{C_0^{\,2}}{C^2} e^{\frac{2M_G}{r}} - \frac{1}{r^2} \equiv 0 \quad \therefore\ \frac{1}{r^2} = -\frac{c^2}{C^2} + \frac{C_0^{\,2}}{C^2} e^{\frac{2M_G}{r}} \quad \text{and}$$

$$\text{(ii)}\quad \frac{d^2\frac{1}{r}}{d\Phi^2} = M_G \frac{C_0^{\,2}}{C^2} e^{\frac{2M_G}{r}} - \frac{1}{r} \equiv 0 \quad \therefore \frac{1}{r} = M_G \frac{C_0^{\,2}}{C^2} e^{\frac{2M_G}{r}} .$$

Therefore $\dfrac{1}{r^2} = -\dfrac{c^2}{C^2} + \dfrac{1}{r M_G}$, $\dfrac{c^2}{C^2} r^2 - \dfrac{1}{M_G} r + 1 = 0.$ (6)

$$\frac{c^2}{\left(r\frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)^2} r^2 - \frac{1}{M_G} r + 1 = 0 \quad , \quad \left(\frac{v}{c}\right)^2 = \frac{M_G}{r}. \tag{7}$$

Moreover, we solve the equation (7). Then,

$$r = \frac{\frac{1}{M_G} \pm \sqrt{\frac{1}{M_G^{\,2}} - 4\frac{c^2}{C^2}}}{2\frac{c^2}{C^2}}, \quad 4\frac{c^2}{C^2} < \frac{1}{M_G^{\,2}} \quad \text{(from the discriminant)}.$$

Therefore, the minimum of this radius and the maximum velocity are as follows:

When , $\dfrac{c}{C} \left(= \dfrac{c}{r\frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}\right) = \dfrac{1}{2M_G}$.

And then $r = \dfrac{\frac{1}{M_G}}{2\frac{c^2}{C^2}} = \dfrac{\frac{1}{M_G}}{2(\frac{1}{2M_G})^2} = 2M_G$ and $v = \dfrac{c}{\sqrt{2}}$.

We calculate the Planet values $\{ r_p u_p \}$of Venus, Earth and Mars as follows:

1) The orbital speed of Venus is $u_p = 3.5020\times10^4{}_{[\mathrm{m/s}]}$ and the Planet radius is $r_p = 1.08204\times10^{11}{}_{[m]}$, then the value of moment is $r_p u_p = 3.78930\times10^{15}{}_{[\mathrm{m^2/s}]}$.

2) The orbital speed of Earth is $u_p = 2.9783\times10^4{}_{[\mathrm{m/s}]}$ and the planet radius is $r_p = 1.49598\times10^{11}{}_{[m]}$, then the value of moment is $r_p u_p = 4.45548\times10^{15}{}_{[\mathrm{m^2/s}]}$.

3) The orbital speed of Mars is $u_p = 2.4128\times10^4{}_{[\mathrm{m/s}]}$ and the planet radius is $r_p = 2.27942\times10^{11}{}_{[m]}$, then the value of moment is $r_p u_p = 5.49978\times10^{15}{}_{[\mathrm{m^2/s}]}$.

By 1), 2) and 3) in order to take out a better point, although there is no basis in particular. But we take the value $(mean) = \dfrac{\frac{Venus + Earth}{2} + Mars}{2} = \underline{\underline{4.81}}109\times10^{15}{}_{[\mathrm{m^2/s}]}$ (no reason).

This formula is no reason, but this is near the value $4.75\times10^{15}{}_{[\mathrm{m^2/s}]}$ by a method of least squares to resonance among Mercury to Pluto. Table 1 shows ratio of the resonance (or the moment) point for the planet.

Table 1 Ratio of the resonance (or the moment) point for the planet.

	Mercury	(mean $r_1 \cdot u_1$)	Jupiter	Saturn	Uranus	Neptune	Pluto
Orbital radius $r_{p[m]}$	5.79×10^{10}	(1.74×10^{11})	7.78×10^{11}	1.43×10^{12}	2.88×10^{12}	4.50×10^{12}	5.92×10^{12}
Orbital speed $u_{p[m/s]}$	4.79×10^{4}	(2.76×10^{4})	1.31×10^{4}	9.64×10^{3}	6.79×10^{3}	5.43×10^{3}	4.74×10^{3}
Moment $r_p \cdot u_p$	2.77×10^{15}	$\underline{\underline{4.81}}\times10^{15}$	1.02×10^{16}	1.38×10^{16}	1.96×10^{16}	2.44×10^{16}	2.81×10^{16}
Ratio of Moment	0.58	1	2.11	2.86	4.06	5.08	5.82

The value(mean) of the set (Venus, Earth, Mars) is 1, then the value of Jupiter , Saturn, Uranus, Neptune ,Pluto is about 2, 3, 4, 5, 6 respectively.

And the mercury is very closed to Sun, therefore its orbit is elliptic and its rotation is affected in revolution. And the planet does not jump easily by the excitation like the electron. Therefore, the other planet occupied its position.

2. The Orbit Equation

The metric is $ds^2 = -dct^2 + dr^2 + r^2(\sin^2\theta d\varphi^2 + d\theta^2)$.

Fig. 2 shows the sphere type. We consider the two-body problem concerned with the nuclear and the electron as in one hydrogen atom. It is assumed that the electron moves on fixed surface. Therefore, we put $\theta = \frac{\pi}{2} - i\Omega$. Ω is a parameter that relates to the angle of rotation on the orbit.

Then the metric is

$$ds^2(= -dc\tau^2) = -dct^2 + dr^2 + r^2(\cosh^2\Omega d\varphi^2 - d\Omega^2)\ (<0) \tag{8}$$

and the polar coordinate is (t, r, Ω, φ).

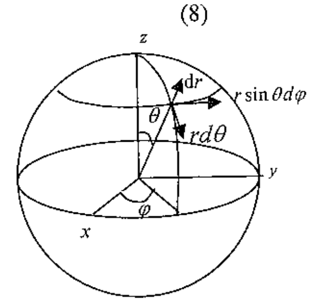

Fig. 2. The sphere type.

We change the situation of the electron to the planet.

Then we get the equation of Kepler's type.

$$\begin{cases} m_0 c\dfrac{dct}{d\tau} = m_0 c C_0 e^{\frac{M_G}{r}} \cdots \text{(the conservation of energy)} \\ \dfrac{d^2}{d\tau^2}(r\sinh\Omega) = -(\dfrac{M_G}{r^2}\dfrac{dct}{d\tau})(\tanh\Omega - r\cosh\Omega\dfrac{d\varphi}{dct})\cosh\Omega(\dfrac{dct}{d\tau}) \\ \qquad\qquad \cdots \text{(the structure of space)} \\ r^2\{(r\cosh\Omega\dfrac{d\varphi}{d\tau})^2 - (r\dfrac{d\Omega}{d\tau})^2\} = C^2 \cdots \text{(the law of equal areas)} \\ r^2\cosh\Omega\dfrac{d\varphi}{d\tau} = C\cosh\Theta(\geqq 0), \quad r\dfrac{d\Omega}{d\tau} = -C\sinh\Theta \\ \Theta = -\int(\dfrac{M_G}{r^2}\dfrac{dct}{d\tau} - \sinh\Omega\dfrac{d\varphi}{d\tau})d\tau \cdots\cdots \text{(the internal rotation)} \end{cases}$$

Fig. 3 shows the anti-de sitter type.

$$\begin{cases} ct = ct \\ x = r\cosh\Omega\cos\varphi \\ y = r\cosh\Omega\sin\varphi \\ iz = ir\sinh\Omega \end{cases}$$

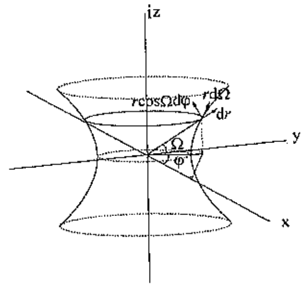

Fig. 3. The anti-de sitter type.

It is proposed that the planet have gravity for the sun and Earth. Where C_0 is the speed constant for the energy function and C is the area speed constant.

The main equation is

$$\left(\frac{dr}{dc\tau}\right)^2 = \left(\frac{dct}{dc\tau}\right)^2 - r^2\left(\left(\cosh\Omega\frac{d\varphi}{dc\tau}\right)^2 - \left(\frac{d\Omega}{dc\tau}\right)^2\right) - 1$$

$$= \left(\frac{C_0}{c}e^{\frac{M_G}{r}}\right)^2 - \left(\frac{C}{cr}\right)^2 - 1, \quad M_G = \frac{GM}{c^2} = 1476.69197[\text{m}] \tag{9}$$

3. The relativistic Mercury orbit

The information of Mercury is $r_1 = 4.6001272\text{x}10^{10}$ m, aphelion is $r_2 = 6.9817079\text{x}10^{10}$ m and eccentricity is $e = 0.20563069$.

Therefore, $\frac{1}{r} = \frac{1}{r_0}(1 + e\cos(n_f\theta))$ $\therefore \frac{1}{r_1} = \frac{1+e}{r_0}$, $\frac{1}{r_2} = \frac{1-e}{r_0}$ $\therefore \frac{1}{r_1} + \frac{1}{r_2} = \frac{2}{r_0}$, $\frac{1}{r_1} - \frac{1}{r_2} = \frac{2e}{r_0}$.

Then $r_0 = 5.5460545\text{ x}10^{10}$ m which is the balanced point of force.

Therefore $r = \dfrac{r_0}{1 + e\cos(n_f\theta)} = \dfrac{5.5460545\times 10^{10}}{1 + 0.20563051\cos(n_f\theta)}$, n_f is very close to 1.

By the value $r_1 = 4.6001272\text{ x}10^{10}$ m and $r_2 = 6.9817079\text{ x}10^{10}$ m,

the solution of $\dfrac{C^2}{r_1^2} - C_0^2 e^{2\frac{M_G}{r_1}} + c^2 = 0$ and $\dfrac{C^2}{r_2^2} - C_0^2 e^{2\frac{M_G}{r_2}} + c^2 = 0$ is

$C = 2.7130495792197815\times 10^{15}{}_{[\text{m/s}]}$ and $C_0 = 2.997924541776255\times 10^{8}{}_{[\text{m/s}]}$.

3.1. The solution of main equation

We deform the main equation, we get the orbit equation for $\frac{1}{r}$ is

$$\left(\frac{d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)^2 = \left(\frac{dr}{r^2 e^{-\frac{M_G}{r}}d\Phi}\right)^2 = \left(\frac{C_0^{\ 2}}{C^2}e^{\frac{2M_G}{r}} - \frac{1}{r^2} - \frac{c^2}{C^2}\right)e^{\frac{2M_G}{r}}, d\theta = e^{-\frac{M_G}{r}}d\Phi.$$

And the orbit equation for r is

$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{dr}{\underline{\underline{e^{-\frac{M_G}{r}}d\Phi}}}\right)^2 = r^2(-1 + r^2\frac{C_0^{\ 2}}{C^2}e^{\frac{2M_G}{r}} - r^2\frac{c^2}{C^2})e^{2\frac{M_G}{r}}, d\theta = \underline{\underline{e^{-\frac{M_G}{r}}d\Phi}}.$$

$$\fallingdotseq r^2\left(-1 + r^2\left(\frac{C_0}{C}\left(1 + \frac{M_G}{r}\right)\right)^2 - r^2\left(\frac{c}{C}\right)^2\right)\left(1 + \frac{M_G}{r}\right)^2 \tag{10}$$

$$= \left(1 + M_G^{\ 2}\left(\frac{c}{C}\right)^2\right)(r + M_G)^2 - 2M_G\left(\frac{c}{C}\right)^2(r + M_G)^3 - \left(\frac{C_0^{\ 2} - c^2}{C^2}\right)(r + M_G)^4.$$

We solve this equation.

$$r = --1476.6919360418137(\fallingdotseq M_G) + \frac{\underline{5.546054}757462805\times 10^{10}}{1 + \underline{\underline{0.2056306}}0089637278\sin(1.0000000133129952\theta)}.$$

<u>This solution is expressed by sinθ not by the cosθ.</u> And this orbit is not elliptic and the perihelion is delay.

And this general formula, $r=-M_G+\dfrac{xM_G}{1+e\sin(n_f\theta)}$. Therefore, we get the differential equation.

$$r'[\theta]^2=n_f{}^2(r+M_G)^2-2\frac{n_f{}^2}{xM_G}(r+M_G)^3-\frac{(e^2-1)n_f{}^2}{(xM_G)^2}(r+M_G)^4.$$

We compared this differential equation and the equation (10).

We get the index (a) $x=\dfrac{C^2}{c^2M_G{}^2}+1$, (b) $n_f=\sqrt{\dfrac{x}{x-1}}$, (c) $e=\sqrt{x\left(\dfrac{C_0{}^2}{c^2}-1\right)+1}$.

The index (b) $n_f=\sqrt{\dfrac{x}{x-1}}$ means that when orbital radius is near the center, gravity (the tensely) will be strong more and be dragged to its tensely hard more. Therefore, a perihelion point in an orbit will show early, and a pole is shifting to the back consequently.

3.2. The elliptic orbit contained the main equation

We deform the main equation to the elliptic differential equation, then we get

$$\left(\frac{\mathrm{d}\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)^2=\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\right)e^{2\frac{M_G}{r}}$$

$$=\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}(1+2\frac{M_G}{r}+2\frac{M_G{}^2}{r^2}+\cdots)-\frac{1}{r^2}\right)(1+2\frac{M_G}{r}+2\frac{M_G{}^2}{r^2}+\cdots)$$

$$=\frac{C_0{}^2-c^2}{C^2}+M_G\left(\frac{4C_0{}^2-2c^2}{C^2}\right)\frac{1}{r}-\left(1-M_G{}^2\frac{8C_0{}^2-2c^2}{C^2}\right)\frac{1}{r^2}+\cdots.$$

Then the elliptic orbit equation for r is

$$\frac{\mathrm{d}r}{\mathrm{d}\theta}=\frac{\mathrm{d}r}{e^{-\frac{M_G}{r}}\mathrm{d}\Phi}=-r\sqrt{\frac{C_0{}^2-c^2}{C^2}r^2+M_G\left(\frac{4C_0{}^2-2c^2}{C^2}\right)r-\left(1-M_G{}^2\frac{8C_0{}^2-2c^2}{C^2}\right)} \tag{11}$$

Then the solution is

$$r=\frac{5.5460540066287445\times10^{10}}{1+0.2056307973976857\sin(0.9999999201220280\theta)}.$$

This orbit is elliptic and the perihelion is advance. And this value is

$$360_{[deg]}\times60^2\left(\frac{1}{0.999999920122028}-1\right)\times415_{(time)}=42.96157181_{[s]}.$$

Where $0.9999999201220280\theta=2\pi$, $\theta=\dfrac{2\pi}{0.999999920122028_{[rad]}}$ is one cycle.

3.3. The perihelion shift

We have the next question. Why does Mercurial advance of perihelion move ahead? Because the area between perihelions to next perihelion of two orbit is equal. And then the time interval is the same. When we get the next relation:

$$\int_0^{2\pi/x_f}\left(\frac{5.5460540066287445\times10^{10}}{1+0.2056307973976857\sin(x_f\theta)}\right)^2\mathrm{d}\theta$$

$$=\int_0^{2\pi/1.0000000133129952}\left(-1476.6919360418137+\frac{5.546054757462805\times10^{10}}{1+0.2056306008963727 8\sin(1.0000000133129952\theta)}\right)^2\mathrm{d}\theta.$$

We get the value $x_f=0.9999999201220279$ which is very close to 0.999999920122028 (elliptic)

This means that the polar-to-polar time interval is the same value. And under the setting which is an elliptic, the location of the pole will slip, see and advance towards the front.

4. The vending of the light

We use the radius of the sun $R_\odot=6.955\times10^8m$ as the perihelion r_1 of light.

And we take the speed v_1 very close to the light speed. Then the area speed constant C and the speed constant for energy function C_0 diverge to infinity.

Where $\left(r\dfrac{\mathrm{d}\Phi}{\mathrm{d}\tau}=\right)\dfrac{v_1}{\sqrt{1-\left(\frac{v_1}{c}\right)^2}}=\dfrac{C}{r_1}$ and $\left(\dfrac{\mathrm{d}ct}{\mathrm{d}\tau}=\right)\dfrac{c}{\sqrt{1-\left(\frac{v_1}{c}\right)^2}}=C_0e^{\frac{M_G}{r_1}}$.

But the ratio of two constants is converge, that is,

when speed v_1 close to $c(light\ speed)$, then $\dfrac{C}{C_0}=r_1\dfrac{v_1}{c}e^{\frac{M_G}{r_1}}$ close to $R_\odot e^{\frac{M_G}{R_\odot}}\fallingdotseq R_\odot$.

Then the elliptic orbit equation (for r)

$$\frac{\mathrm{d}r}{e^{-\frac{M_G}{r}}\mathrm{d}\Phi}=-r\sqrt{\frac{C_0{}^2-c^2}{C^2}r^2+M_G\left(\frac{4C_0{}^2-2c^2}{C^2}\right)r-\left(1-M_G{}^2\frac{8C_0{}^2-2c^2}{C^2}\right)}. \tag{12}$$

It is converge to

$$\frac{\mathrm{d}r}{\mathrm{d}\theta}=\frac{\mathrm{d}r}{e^{-\frac{M_G}{r}}\mathrm{d}\Phi}=-r\sqrt{\frac{1}{R_\odot{}^2}r^2+M_G\left(\frac{4}{R_\odot{}^2}\right)r-\left(1-M_G{}^2\frac{8}{R_\odot{}^2}\right)}.$$

Then the orbit is hyperbolic and

$$r = \frac{1.64021\times10^{14}}{1+235662\sin(0.999999999981994\theta)}.$$

Fig. 4 shows the curve of light.

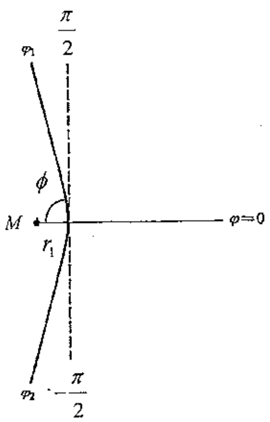

Fig.4.The curve of light

We compare the elliptic(hyperbolic) orbit

$$\frac{1}{r} = \frac{\dfrac{2M_G}{R_\odot^{\ 2}}}{1-\dfrac{8M_G^{\ 2}}{R_\odot^{\ 2}}}\left(1+e\sin\left(\sqrt{1-\frac{8M_G^{\ 2}}{R_\odot^{\ 2}}}\cdot\theta\right)\right),\quad d\theta = e^{\frac{M_G}{r}}d\Phi.$$

$$r_0 = \frac{1-\dfrac{8M_G^{\ 2}}{R_\odot^{\ 2}}}{\dfrac{2M_G}{R_\odot^{\ 2}}} = 1.64021\times10^{14},$$

$$n_f = \sqrt{1-\frac{8M_G^{\ 2}}{R_\odot^{\ 2}}} = 0.999999999981994,$$

$$R_\odot(=r_1) = \frac{1.64021\times10^{14}}{1+235662} = 6.95997\times10^{8},\quad e = \frac{1-\dfrac{8M_G^{\ 2}}{R_\odot^{\ 2}}}{\dfrac{2M_G}{R_\odot}}\left(=\frac{r_0}{r_1}\right)-1 \fallingdotseq \frac{R_\odot}{2M_G} = 235662.$$

The slope of asymptote is $\tan\phi = \sqrt{e^2-1} \fallingdotseq e = 235662$.

Therefore, the vending of light is

$$2\left(\frac{\pi}{2}-\phi\right) \fallingdotseq 2\tan\left(\frac{\pi}{2}-\phi\right) = \frac{2}{\tan\phi} = \frac{2}{235662} = 8.48674\times10^{-6}[\mathrm{rad}] = \underline{1.75}051''.$$

5. Conclusion

The movement of Planet is similar to one of electron. And its equation is those that have change the R_0 (in the electromagnetic) to the $M_G = \dfrac{GM}{c^2}$ (in the gravity).

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