~~重力と電磁気力~~

The Gravitational Force and the Electromagnetic Force*

Yoshio TAKEMOTO**, Seishu SHIMAMOTO***

Department of Mechanical and Electrical Engineering, School of Engineering,
Nippon Bunri University

Abstract

This paper provides the matrix which contains the gravitation and the electromagnetic field. We explain that the similarity or difference between the gravitation forces and electromagnetic force.

1. Introduction

1.1. Definition of the imaginary charge and its derivatives

The magnetic charge m is a pure imaginary part of the complex charge as $e-\mathrm{i}m$, then its potential (1), magnetic field (2), Maxwell equation (3) and the Coulomb-Lorentz force (4) are

$$ {}^{+}\begin{bmatrix} -\mathrm{i}\phi & \\ & -\mathrm{i}c\mathbf{A}(=\mathbf{0}) \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} -\mathrm{i}m & \\ & \mathbf{0} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \dfrac{1}{|\mathbf{r}|} & \\ & \mathbf{0} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} -\mathrm{i}\dfrac{m}{|\mathbf{r}|} & \\ & \mathbf{0} \end{bmatrix}^{+} . \tag{1} $$

$$ \begin{aligned} {}^{-}\begin{bmatrix} -\mathrm{i}B_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} &= {}^{-}\begin{bmatrix} \partial ct & \\ & -\partial\mathbf{r} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} -\mathrm{i}\,\phi & \\ & -\mathrm{i}c\mathbf{A} \end{bmatrix}^{+} \\ &= {}^{-}\begin{bmatrix} -\mathrm{i}\dfrac{\partial\phi}{\partial ct}+\mathrm{i}\,div\,c\mathbf{A} & \\ & -\mathrm{i}\dfrac{\partial c\mathbf{A}}{\partial ct}+\mathrm{i}\,\mathbf{grad}\,\phi+\mathbf{rot}\,c\mathbf{A} \end{bmatrix}^{+} . \end{aligned} \tag{2} $$

$$ {}^{+}\begin{bmatrix} -\mathrm{i}m & \\ & \mathbf{0} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \partial ct & \\ & \partial\mathbf{r} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} -\mathrm{i}B_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} . \tag{3} $$

$$
{}^{-}\begin{bmatrix} F_t & \\ & \mathbf{F} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} -\mathrm{i}E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} \overline{{}^{-}\begin{bmatrix} e-\mathrm{i}m & \\ & \mathbf{0} \end{bmatrix}^{-}} , \tag{4}
$$

where $\overline{e-\mathrm{i}m} = e+\mathrm{i}m$ means a complex conjugate.

1.2. The matrix expression of the mass and its fields like the magnetic charge

It is the same as that of the magnetic charge m. We put the mass M as $\mathrm{i}M$ which is a pure imaginary number, because the two masses has the power of absorption. Then its potential (5), (magnetic) field (6), Maxwell equation (7) and the Coulomb-Lorentz force (8) are

$$
{}^{+}\begin{bmatrix} \mathrm{i}\phi_g & \\ & -\mathrm{i}\mathbf{A}_g(=\mathbf{0}) \end{bmatrix}^{+} = G\,{}^{+}\begin{bmatrix} \mathrm{i}M & \\ & \mathbf{0} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \dfrac{1}{|\mathbf{r}|} & \\ & \mathbf{0} \end{bmatrix}^{+} = G\,{}^{+}\begin{bmatrix} \mathrm{i}\dfrac{M}{|\mathbf{r}|} & \\ & \mathbf{0} \end{bmatrix}^{+} . \tag{5}
$$

$$
\begin{aligned}
{}^{-}\begin{bmatrix} \mathrm{i}B_{gt} & \\ & \mathbf{E}_g-\mathrm{i}c\mathbf{B}_g \end{bmatrix}^{+} &= {}^{-}\begin{bmatrix} \partial ct & \\ & -\partial\mathbf{r} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \mathrm{i}\,\phi_g & \\ & -\mathrm{i}c\mathbf{A}_g \end{bmatrix}^{+} \\
&= {}^{-}\begin{bmatrix} \mathrm{i}\dfrac{\partial\phi_g}{\partial ct}+\mathrm{i}\,div\,c\mathbf{A}_g & \\ & -\mathrm{i}\dfrac{\partial c\mathbf{A}_g}{\partial ct}-\mathrm{i}\,\mathbf{grad}\,\phi_g+\mathbf{rot}\,c\mathbf{A}_g \end{bmatrix}^{+} .
\end{aligned} \tag{6}
$$

$$
{}^{+}\begin{bmatrix} \mathrm{i}M & \\ & \mathbf{0} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \partial ct & \\ & \partial\mathbf{r} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \mathrm{i}B_{gt} & \\ & \mathbf{E}_g-\mathrm{i}c\mathbf{B}_g \end{bmatrix}^{+} . \tag{7}
$$

$$
{}^{-}\begin{bmatrix} F_t & \\ & \mathbf{F} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} \mathrm{i}B_{gt} & \\ & \mathbf{E}_g-\mathrm{i}c\mathbf{B}_g \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \mathrm{i}M & \\ & \mathbf{0} \end{bmatrix}^{-} , \quad \text{this is not} \quad \overline{{}^{-}\begin{bmatrix} \mathrm{i}M & \\ & \mathbf{0} \end{bmatrix}^{-}} , \tag{8}
$$

where $\overline{\mathrm{i}M} = -\mathrm{i}M$ means a complex conjugate.

2. The matrix which contained the electric charge and the mass simultaneously

2.1 The electric charge and the mass in the same matrix

We put the electric charge e and the magnetic charge m as $(e-\mathrm{i}m)1_t$ which is the time component and the mass $M(1_x, 1_y, 1_z)$ which is the space component, which means that the magnetic charge is on the complex conjugate and the mass is on the space conjugate as follows:

$$(\vec{\mathbf{e}}-\mathrm{i}\overrightarrow{\mathbf{m}})+\overrightarrow{\mathbf{M}} = {}^{+}\begin{bmatrix} k_0(e-\mathrm{i}m)1_t & \\ & -GM(1_x,1_y,1_z) \end{bmatrix}^{+}$$

$$= {}^{+}\begin{bmatrix} k_0(e-\mathrm{i}m)\left(\begin{array}{c|ccc} \underline{1} & \underline{0} & \underline{0} & \underline{0} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) & \\ & -GM\left(\left(\begin{array}{c|ccc} \underline{0} & \underline{1} & \underline{0} & \underline{0} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 0 & 0 & -\mathrm{i} & 0 \end{array}\right), \left(\begin{array}{c|ccc} \underline{0} & \underline{0} & \underline{1} & \underline{0} \\ 0 & 0 & 0 & -\mathrm{i} \\ 1 & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \end{array}\right), \left(\begin{array}{c|ccc} \underline{0} & \underline{0} & \underline{0} & \underline{1} \\ 0 & 0 & \mathrm{i} & 0 \\ 0 & -\mathrm{i} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)\right) \end{bmatrix}^{+}.$$

Then we can use the same standard for each unit of the charge $e-\mathrm{i}m$ and the mass M. For simplicity, we take the complex charge as $q = e-\mathrm{i}m$ and "the charge and mass" as $\vec{\mathbf{q}}+\overrightarrow{\mathbf{M}}$ $= {}^{+}\begin{bmatrix} k_0 q 1_t & \\ & -GM(1_x,1_y,1_z) \end{bmatrix}^{+}$. And its potential (9), field (10), the Maxwell equation (11) are

$${}^{+}\begin{bmatrix} \phi_{ge} & \\ & -\mathrm{c}\mathbf{A}_{ge} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} k_0 q 1_t & \\ & -GM(1_x,1_y,1_z) \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \dfrac{1}{|\mathbf{r}|} & \\ & \mathbf{0} \end{bmatrix}^{+}$$

$$= {}^{+}\begin{bmatrix} \dfrac{k_0 e}{|\mathbf{r}|}1_t & \\ & -\dfrac{GM}{|\mathbf{r}|}(1_x,1_y,1_z) \end{bmatrix}^{+}. \tag{9}$$

$${}^{-}\begin{bmatrix} E_{get} & \\ & \mathbf{E}_{ge}-\mathrm{i}\mathrm{c}\mathbf{B}_{ge} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \partial \mathrm{c}t & \\ & -\partial\mathbf{r} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \phi_{ge} & \\ & -\mathrm{c}\mathbf{A}_{ge} \end{bmatrix}^{+}$$

$$= {}^{-}\begin{bmatrix} \dfrac{\partial\phi_{ge}}{\partial \mathrm{c}t} + div\,\mathrm{c}\mathbf{A}_{ge} & \\ & -\dfrac{\partial \mathrm{c}\mathbf{A}_{ge}}{\partial \mathrm{c}t} - \mathbf{grad}\,\phi_{ge} - \mathrm{i}\mathbf{rot}\,\mathrm{c}\mathbf{A}_{ge} \end{bmatrix}^{+}. \tag{10}$$

$$\overrightarrow{\mathbf{q}_0}+\overrightarrow{\mathbf{M}_0} = {}^{+}\begin{bmatrix} q1_t & \\ & -M(1_x,1_y,1_z) \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \partial \mathrm{c}t & \\ & \partial\mathbf{r} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} E_{get} & \\ & \mathbf{E}_{ge}-\mathrm{i}\mathrm{c}\,\mathbf{B}_{ge} \end{bmatrix}^{+}. \tag{11}$$

2.2. Maxwell Equation and the temporary unit

(i) The temporary unit of the electron

By the Coulomb force $F=\dfrac{k_0 eq}{r^2}$, we temporarily define the charge which has the capability to generate an electric potential as $e_{w[Cw]}$, $q_{w[Cw]}$ instead of e , q and $F=\dfrac{k_0 eq}{r^2}=\dfrac{e_w q_w}{r^2}$.

By the equation of motion $F=m_e\alpha=m_e\dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$ (Newton), we temporarily define the mass of the charge which has the difficulty $m=m_{e[kgi]}$ of moving to the electromagnetic field.

More specifically, the symbol $e_{w[Cw]}$ is the physical meaning that the quantity $e_{w[kgw]}$ can accelerate the mass $m_{e[kgi]}$ by the value $\dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$. In this case the units kgi and kg are same and by the relation $\dfrac{{e_w}^2}{1^2}=F_{[kgi\cdot m/s^2]}=\dfrac{k_{0[kgi\cdot m^3/(C^2\cdot s^2)]}e^2}{1^2}$, we get $e_{w\,[Cw=kgi^{\frac{1}{2}}\cdot m^{\frac{3}{2}}/s]}=\sqrt{k_{0[kgi\cdot m^3/(C^2\cdot s^2)]}}e_{[C]}$.

(ii) The temporary unit of the mass

By the Universal gravitation of Newton $F=-\dfrac{GMm}{r^2}$, We temporarily define the mass which has the capability to generate a gravitational potential as $M_{w[kgw]}$, $m_{w[kgw]}$ instead of M , m and $F=-\dfrac{GMm}{r^2}=-\dfrac{M_w m_w}{r^2}$.

By the equation of motion $F=m\alpha=m\dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$ (Newton) and more we temporarily define the mass of the particle which has the difficulty $m=m_{i[kgi]}$ of moving to the gravitational field.

More specifically, the symbol $m_{w[kgw]}$ is the physical meaning that the quantity $m_{w[kgw]}$ can accelerate the mass $m_{i[kgi]}$ by the value $\dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$. In this case the units kgi and kg are same, and by the relation $-\dfrac{{m_w}^2}{1^2}=F_{[kgi\cdot m/s^2]}=-\dfrac{G_{[kgi\cdot m^3/(kg^2\cdot s^2)]}m^2}{1^2}$, we get $m_{w\,[kgw=kgi^{\frac{1}{2}}\cdot m^{\frac{3}{2}}/s]}$

$$= \sqrt{G_{[kgi\cdot m^3/(kg^2\cdot s^2)]}} m_{[kg]} .$$

2.3. The Coulomb and the Gravitational Force of "the charge and the mass".

We defined a source $\vec{\mathbf{q}}_w + \overrightarrow{\mathbf{M}_w} = {}^{+}\begin{bmatrix} q_w 1_t & \\ & -M_w(1_x,1_y,1_z) \end{bmatrix}^{+}$,potential ${}^{+}\begin{bmatrix} \phi_{ge} & \\ & -\mathrm{c}\mathbf{A}_{ge} \end{bmatrix}^{+}$

,field ${}^{-}\begin{bmatrix} E_{get} & \\ & \mathbf{E}_{ge} - \mathrm{i}\mathrm{c}\mathbf{B}_{ge} \end{bmatrix}^{+}$ and particle ${}^{-}\begin{bmatrix} u_{ge0} & \\ & \boldsymbol{u}_{ge} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} q'_w 1_t & \\ & M'_w(1_x,1_y,1_z) \end{bmatrix}^{-}$ are as above 2.1.

Furthermore, we expect the Coulomb and the Gravitational force as

$$ {}^{-}\begin{bmatrix} F_t & \\ & \mathbf{F} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} E_{get} & \\ & \mathbf{E}_{ge} - \mathrm{i}\mathrm{c}\mathbf{B}_{ge} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} q'_w 1_t & \\ & M'_w(1_x,1_y,1_z) \end{bmatrix}^{-} . \tag{12}$$

But this is not the same size matrix. Therefore we take the variation method as follows[1)]:

We put ${}^{-}\begin{bmatrix} \frac{E}{\mathrm{c}} & \\ & \mathbf{P} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} \frac{E_0}{\mathrm{c}} & \\ & \mathbf{p} \end{bmatrix}^{-} + {}^{-}\begin{bmatrix} \frac{q_w}{\mathrm{c}}\phi_{ge} & \\ & \frac{M_w}{\mathrm{c}}\mathrm{c}\mathbf{A}_{ge} \end{bmatrix}^{-}$ in the previous paper, then

$$Tr({}^{+}\begin{bmatrix} \delta \mathrm{c}t & \\ & -\delta\mathbf{r} \end{bmatrix}^{+} \left(\frac{\mathrm{d}}{\mathrm{d}\mathrm{c}\tau} {}^{-}\begin{bmatrix} \frac{E}{\mathrm{c}} & \\ & \mathbf{P} \end{bmatrix}^{-} - {}^{-}\begin{bmatrix} E_{get} & \\ & \mathbf{E}_{ge} - \mathrm{i}\mathrm{c}\mathbf{B}_{ge} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} q'_w \frac{u_{ge0}}{\mathrm{c}} & \\ & M'_w \frac{\mathbf{u}_{ge}}{\mathrm{c}} \end{bmatrix}^{-} \right)) = 0 . \tag{13}$$

Concretely when the particle is not move, the potential (9) and field (10) are

$$\phi(\vec{\mathbf{q}},\overrightarrow{\mathbf{M}}) = \frac{1}{r}$$

$$\bullet {}^{+}\begin{bmatrix} q_w \begin{pmatrix} \underline{1}| & \underline{0} & \underline{0} & \underline{0} \\ 0| & 1 & 0 & 0 \\ 0| & 0 & 1 & 0 \\ 0| & 0 & 0 & 1 \end{pmatrix} & \\ & -M_w \left(\begin{pmatrix} \underline{0}| & \underline{1} & \underline{0} & \underline{0} \\ 1| & 0 & 0 & 0 \\ 0| & 0 & 0 & \mathrm{i} \\ 0| & 0 & -\mathrm{i} & 0 \end{pmatrix}, \begin{pmatrix} \underline{0}| & \underline{0} & \underline{1} & \underline{0} \\ 0| & 0 & 0 & -\mathrm{i} \\ 1| & 0 & 0 & 0 \\ 0| & \mathrm{i} & 0 & 0 \end{pmatrix}, \begin{pmatrix} \underline{0}| & \underline{0} & \underline{0} & \underline{1} \\ 0| & 0 & \mathrm{i} & 0 \\ 0| & -\mathrm{i} & 0 & 0 \\ 1| & 0 & 0 & 0 \end{pmatrix} \right) \end{bmatrix}^{+} . \tag{14}$$

$$\mathbf{E}(\vec{\mathbf{q}},\overrightarrow{\mathbf{M}}) = -\frac{1}{r^3}$$

$$\cdot \begin{bmatrix} M_w \begin{pmatrix} 0 & x & y & z \\ x & 0 & \mathrm{i}z & -\mathrm{i}y \\ y & -\mathrm{i}z & 0 & \mathrm{i}x \\ z & \mathrm{i}y & -\mathrm{i}x & 0 \end{pmatrix} & \\ & \left[-q_w x \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + M_w \begin{pmatrix} 0 & 0 & \mathrm{i}z & -\mathrm{i}y \\ 0 & 0 & y & z \\ \mathrm{i}z & -y & 0 & 0 \\ -\mathrm{i}y & -z & 0 & 0 \end{pmatrix}, -q_w y \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + M_w \begin{pmatrix} 0 & -\mathrm{i}z & 0 & \mathrm{i}x \\ -\mathrm{i}z & 0 & -x & 0 \\ 0 & x & 0 & z \\ \mathrm{i}x & 0 & -z & 0 \end{pmatrix}, -q_w z \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + M_w \begin{pmatrix} 0 & \mathrm{i}y & -\mathrm{i}x & 0 \\ \mathrm{i}y & 0 & 0 & -x \\ -\mathrm{i}x & 0 & 0 & -y \\ 0 & x & y & 0 \end{pmatrix} \right] \end{bmatrix}^{+} \quad (15)$$

Additionally, we calculate the above trace (13).

(i) When $\delta \mathbf{r} = 0$ (time component)

$$Tr\left(\frac{\mathrm{d}\frac{E}{\mathrm{c}}}{\mathrm{d}\mathrm{c}\tau}\right) = Tr\left(\left\{E_{get} \cdot q'_w \frac{u_{ge0}}{\mathrm{c}} + (\mathbf{E}_{ge} - \mathrm{i}\mathrm{c}\mathbf{B}_{ge}) \bullet M'_w \frac{\mathbf{u}_{ge}}{\mathrm{c}}\right\}\right) = 0 \ .$$

(ii) When $\delta \mathrm{c}t = 0$ (space component)

$$Tr\left(\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}\mathrm{c}\tau}\right) = Tr\left(E_{get} \cdot M'_w \frac{\mathbf{u}_{ge}}{\mathrm{c}} + (\mathbf{E}_{ge} - \mathrm{i}\mathrm{c}\mathbf{B}_{ge}) \cdot q'_w \frac{u_{ge0}}{\mathrm{c}} - \mathrm{i}(\mathbf{E}_{ge} - \mathrm{i}\mathrm{c}\mathbf{B}_{ge}) \times M'_w \frac{\mathbf{u}_{ge}}{\mathrm{c}}\right) \ .$$

For simplicity we limit the x-direction.

The charge and the mass is $\overrightarrow{\mathbf{q}_w} + \overrightarrow{\mathbf{M}_w} = {}^{+}\begin{bmatrix} q_w 1_t & \\ & -M_w(1_x, 1_y, 1_z) \end{bmatrix}^{+}$

$$= {}^{+}\begin{bmatrix} q_w \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \\ & -M_w \left(\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 0 & 0 & -\mathrm{i} & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 1 & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \mathrm{i} & 0 \\ 0 & -\mathrm{i} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right) \end{bmatrix}^{+} .$$

And the electric and the gravitational field is $\mathbf{E}(\mathbf{q},\mathbf{M})$

$$
= -\frac{1}{r^3}\;{}^{-}\!\begin{bmatrix} M_w\left(\begin{array}{c|ccc} 0 & x & 0 & 0 \\ \hline x & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i}x \\ 0 & 0 & -\mathrm{i}x & 0 \end{array}\right) & \\ & \begin{bmatrix} -q_w\left(\begin{array}{c|ccc} x & 0 & 0 & 0 \\ \hline 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & x \end{array}\right), M_w\left(\begin{array}{c|ccc} 0 & 0 & 0 & \mathrm{i}x \\ \hline 0 & 0 & -x & 0 \\ 0 & x & 0 & 0 \\ \mathrm{i}x & 0 & 0 & 0 \end{array}\right), M_w\left(\begin{array}{c|ccc} 0 & 0 & -\mathrm{i}x & 0 \\ \hline 0 & 0 & 0 & -x \\ -\mathrm{i}x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \end{array}\right) \end{bmatrix} \end{bmatrix}^{+}.
$$

Therefore we calculate the trace

$$
Tr\left({}^{+}\!\begin{bmatrix} \delta ct & \\ & -\delta\mathbf{r} \end{bmatrix}^{+} {}^{-}\!\begin{bmatrix} E_{get} & \\ & \mathbf{E}_{ge}-\mathrm{i}c\mathbf{B}_{ge} \end{bmatrix}^{+} {}^{-}\!\begin{bmatrix} q'_w\dfrac{u_{ge0}}{\mathrm{c}} & \\ & M'_w\dfrac{\mathbf{u}_{ge}}{\mathrm{c}} \end{bmatrix}^{-} \right) = \mathbf{0}\cdot
$$

(The time component)

$$
F_t = Tr\left(\frac{\mathrm{d}\frac{E}{\mathrm{c}}}{\mathrm{d}c\tau}\right) = Tr\left(E_{get}\cdot q'_w\frac{u_{ge0}}{\mathrm{c}} + (\mathbf{E}_{ge}-\mathrm{i}c\mathbf{B}_{ge})\bullet M'_w\frac{\mathbf{u}_{ge}}{\mathrm{c}}\right) = 0\ \cdot
$$

(The space component)

$$
\mathbf{F} = Tr\left(\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}c\tau}\right) = Tr\left(\underline{E_{get}\cdot M'_w\frac{\mathbf{u}_{ge}}{\mathrm{c}}} + \underline{\underline{(\mathbf{E}_{ge}-\mathrm{i}c\mathbf{B}_{ge})\cdot q'_w\frac{u_{ge0}}{\mathrm{c}}}} - \underline{\underline{\underline{\mathrm{i}(\mathbf{E}_{ge}-\mathrm{i}c\mathbf{B}_{ge})\times M'_w\frac{\mathbf{u}_{ge}}{\mathrm{c}}}}}\right)\ ,
$$

where

$$
\underline{Tr\left(E_{get}\cdot M'_w\frac{\mathbf{u}_{gex}}{\mathrm{c}}\right)} = Tr\left(\frac{-M_wM'_w}{\mathrm{c}}\left(\begin{array}{c|ccc} 0 & x & 0 & 0 \\ \hline x & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i}x \\ 0 & 0 & -\mathrm{i}x & 0 \end{array}\right)\left(\begin{array}{c|ccc} 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 0 & 0 & -\mathrm{i} & 0 \end{array}\right)\right) = -4\,\frac{M_wM'_w}{\mathrm{c}}x\ ,
$$

$$
\underline{\underline{Tr\left((\mathbf{E}_{ge}-\mathrm{i}c\mathbf{B}_{ge})_x\cdot q'_w\frac{u_{ge0}}{\mathrm{c}}\right)}} = Tr\left(\frac{q_wq'_w}{\mathrm{c}}\left(\begin{array}{c|ccc} x & 0 & 0 & 0 \\ \hline 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ 0 & 0 & 0 & x \end{array}\right)\left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)\right) = 4\,\frac{q_wq'_w}{\mathrm{c}}x\ ,
$$

$$\underline{\underline{\underline{Tr(-\mathrm{i}(\mathbf{E}_{ge}-\mathrm{ic}\mathbf{B}_{ge})\times M'_w\frac{\mathbf{u}_{ge}}{\mathrm{c}})}}}$$

$$=Tr(\mathrm{i}\frac{M_wM'_w}{\mathrm{c}}\begin{pmatrix}0 & 0 & 0 & \mathrm{i}x\\ 0 & 0 & -x & 0\\ 0 & x & 0 & 0\\ \mathrm{i}x & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}0 & 0 & 0 & 1\\ 0 & 0 & \mathrm{i} & 0\\ 0 & -\mathrm{i} & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix}-\mathrm{i}\frac{M_wM'_w}{\mathrm{c}}\begin{pmatrix}0 & 0 & -\mathrm{i}x & 0\\ 0 & 0 & 0 & -x\\ -\mathrm{i}x & 0 & 0 & 0\\ 0 & x & 0 & 0\end{pmatrix}\begin{pmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & -\mathrm{i}\\ 1 & 0 & 0 & 0\\ 0 & \mathrm{i} & 0 & 0\end{pmatrix})$$

$$=-4\frac{M_wM'_w}{\mathrm{c}}x-4\frac{M_wM'_w}{\mathrm{c}}x\ .$$

Therefore the space component

$$\mathbf{F}=Tr(\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}\mathrm{c}\tau})=Tr(E_{get}\cdot M'_w\frac{\mathbf{u}_{ge}}{\mathrm{c}}+(\mathbf{E}_{ge}-\mathrm{ic}\mathbf{B}_{ge})\cdot q'_w\frac{u_{ge0}}{\mathrm{c}}-\mathrm{i}(\mathbf{E}_{ge}-\mathrm{ic}\mathbf{B}_{ge})\times M'_w\frac{\mathbf{u}_{ge}}{\mathrm{c}})=4(\frac{q_wq'_w}{\mathrm{c}}-\underline{\underline{3}}\frac{M_wM'_w}{\mathrm{c}})$$

As a result, this shows that the universal gravitation has 3 times as the much relation as electromagnetic power; we think that this reason is that the time is one-direction and the space is a three-direction.

3. The planet and atom

3.1. Angular momentum

The hypothesis about electron arrangement (orbit).

"The electron which is not excited is arranged one by one so that it may become a fixed angular momentum (the amount of resonance)."

Example 1 (Hydrogen nucleus H) the ionization energy is $E_H=13.598e\mathrm{V}$

We put the circular orbit radius r_1 of the electron which is not excited. Then the speed is $\frac{v_1}{\mathrm{c}}=\sqrt{\frac{R_0}{r_1}}(=\sqrt{\frac{k_0e^2}{m_e\mathrm{c}^2r_1}})$ by the balance equation $r_1(\frac{v_1}{\mathrm{c}})^2=R_0(=\frac{k_0e^2}{m_e\mathrm{c}^2})$. Therefore the momentum is $m_er_1v_1=m_e\mathrm{c}\sqrt{r_1R_0}(=\sqrt{m_er_1k_0e^2}=\frac{h}{2\pi}=\hbar)$ where $\hbar=1.05457\times10^{-34}{}_{[kg\cdot m^2/s]}$.

Example 2 (Helium nucleus He^+) the ionization energy is $E_{He^+}=54.416e\mathrm{V}(=E_H\times4)$

We put the circular orbit radius $r_1'=\frac{r_1}{2}$, $R_0'=2R_0$ of the electron which is not excited. Then the speed is $\frac{v_1'}{c} = \sqrt{\frac{R_0'}{r_1'}} = 2\sqrt{\frac{R_0}{r_1}} = \frac{2v_1}{c}$ by the balance equation $r_1'(\frac{v_1'}{c})^2 = R_0'(= \frac{2k_0e^2}{m_e c^2})$ $= 2R_0$.

Therefore the momentum is $m_e r_1' v_1' = m_e \frac{r_1}{2}(2v_1) = m_e r_1 v_1 (= \hbar)$.

Example 3 (Lithium atom core Li^{2+}) the ionization energy is $E_{Li^{2+}} = 122.451 e\mathrm{V} (= E_H \times 9)$

We put the circular orbit radius $r_1'' = \frac{r_1}{3}$, $R_0'' = 3R_0$ of the electron which is not excited. Then the speed is $\frac{v_1''}{c} = \sqrt{\frac{R_0''}{r_1''}} = 3\sqrt{\frac{R_0}{r_1}} = \frac{3v_1}{c}$ by the balance equation $r_1''(\frac{v_1''}{c})^2 = R_0''(= \frac{3k_0e^2}{m_e c^2})$ $= 3R_0$.

Therefore the momentum is $m_e r_1'' v_1'' = m_e \frac{r_1}{3}(3v_1) = m_e r_1 v_1 (= \hbar)$.

Moreover, it can be concluded that the point $\frac{r_1}{n}$ of resonating according to the strength of an electric field is near in inverse proportion to the number n of the protons in a core. Thereby, a hypothesis can be prepared, saying, "The angular momentum of non-excited electron is constant which was not depended on the number n of the protons in a core."

3.1.1. The electron around the atomic nucleus

Here, we define the resonance value $k = \frac{m_e r_1 v_1}{e_{[C]} \cdot 1_{[C]}} \cong 6.58205 \times 10^{-16}{}_{[kg \cdot m^2/(C^2 \cdot s)]}$ as the point of the resonance instead of angular momentum $\hbar = m_e r_1 v_1$ in the hydrogen atom, moreover this resembles the resonance of the whistle, and the resonating point becomes near in proportion to the value of a central electric charge q.

3.1.2. The planet and the satellite for solar system

In the case of the planet which goes around the Sun, the resonating point becomes far in proportion to the mass M of a central star, therefore we define the resonance value $K_s = \frac{m_i r_1 v_1}{m \cdot M}{}_{[m^2/(kg \cdot s)]}$ (where m is the mass and m_i is the inertial mass), as the point of the resonance instead of angular momentum, moreover this resembles the relation of the size and pitch of a drum.

3.1.3. The calculation for the resonance value K

It takes into consideration that universal gravitation has 3 times as much relation as electromagnetic power in the last of the section 2. We take a standard value of the resonance $K_s = 3_{[C^2/kg^2]} \times k = 1.97461 \times 10^{-15}{}_{[m^2/(kg \cdot s)]}$ in the planet and each point of the resonance is measured as follows:

Example 4 The resonance value $K_0 = \dfrac{rv}{M}$ of the planet around the Sun

We calculate the resonance values of the Venus, the Earth and the Mars as follows;

(i) The orbital speed of the Venus is $v = 3.5020 \times 10^4{}_{[m/s]}$ and the distance between the Venus and the Sun is $r = 1.08204 \times 10^{11}{}_{[m]}$.

Then we get the value $K_0 = \dfrac{rv}{M_{[kg]}} = 1.90516 \times 10^{-15}{}_{[m^2/(kg \cdot s)]}$.

(ii) The orbital speed of the Earth is $v = 2.9783 \times 10^4{}_{[m/s]}$ and the distance between the Earth and the Sun is $r = 1.49598 \times 10^{11}{}_{[m]}$. Then we get the value $K_0 = \dfrac{rv}{M_{[kg]}} = 2.24013 \times 10^{-15}{}_{[m^2/(kg \cdot s)]}$.

(iii) The orbital speed of the Mars is $v = 2.4128 \times 10^4{}_{[m/s]}$ and the distance between the Mars and the Sun is $r = 2.27942 \times 10^{11}{}_{[m]}$. Then we get the value $K_0 = \dfrac{rv}{M_{[kg]}} = 2.76518 \times 10^{-15}{}_{[m^2/(kg \cdot s)]}$.

By (1), (2) and (3), the standard value of the resonance $K_s = 1.97461 \times 10^{-15}{}_{[m^2/(kg \cdot s)]}$ is between the Venus and the Earth.

In order to take out a better point, although there is no basis in particular. But we take the value $K_m(=mean) = \dfrac{\dfrac{Venus + Earth}{2} + Mars}{2} = 2.42 \times 10^{-15}{}_{[m^2/(kg \cdot s)]}$ (no reason), $r_1 = K_m{}^2 \dfrac{M}{G}$,

$v_1 = \dfrac{G}{K_m}$ as a point of the resonance.

This formula is no reason, but this is near the value $2.23\times10^{-15}{}_{[m^2/(kg\cdot s)]}$ by a method of least squares to resonance among Jupiter to Pluto. Therefore we get the next table 1.

Table 1

Ratio of the resonance point for the planet.

	Mercury	**(mean)**	Jupiter	Saturn	Uranus	Neptune	Pluto
Orbital radius[m]	5.79E+10	(1.75E+11)	7.78E+11	1.43E+12	2.88E+12	4.50E+12	5.92E+12
Orbital speed [m/s]	4.79E+04	(2.76E+04)	1.31E+04	9.64E+03	6.79E+03	5.43E+03	4.74E+03
Resonance[m^2/(kg ·s)]	1.39E-15	2.42E-15	5.11E-15	6.92E-15	9.82E-15	1.23E-14	1.41E-14
Ratio of resonance	0.58	1	2.11	2.86	4.06	5.08	5.82

The value of the set (Venus, Earth, Mars) is 1, then the value of Jupiter , Saturn, Uranus, Neptune ,Pluto is about 2, 3, 4, 5, 6 respectively.

We compare two resonance value the mean value and the standard value, and its ratio is $R(=\dfrac{m}{m_i}) = \dfrac{mean}{K_s} = \dfrac{2.42\times10^{-15}}{1.97\times10^{-15}} = 1.225_{[-]}$.

This is equivalent to having estimated the weight of the central star R times greatly. And the mercury is very closed to the Sun, therefore its orbit is elliptic and its rotation is affected in revolution.

Example 5 The resonance value $K_0 = \dfrac{rv}{M}$ of the satellite around the planet

We calculate the resonance values of a satellite around the planet as follows;

By two formula $K = \dfrac{r_1 v_1}{M}$ (resonance) and $r_1 v_1^{\,2} = GM$ (balance equation in the circle orbit), we get the expected the minimum orbital radius of the satellite $r_{1[m]} = K^2{}_{[m^4/(kg^2\cdot s^2)]} \dfrac{M_{[kg]}}{G_{[m^3/(kg\cdot s)]}}$.

For example, the case of the Earth and the moon, the orbital radius of the moon is 3.84×10^8 m, the orbital speed is 1.018×10^3 m/s and the Earth mass is 5.977×10^{24} kg. Therefore the position of the satellite expected is $r_1 = K^2 \dfrac{R\cdot M}{G} \fallingdotseq 4.27614\times10^5 m$, where the mass of the central star is being corrected in R double here. This value $r_1 \fallingdotseq 4.27614\times10^5 m$ means that it's inside the equatorial radius 6378000 m of the Earth. At other planets, the expected radius r_1 is inside the equatorial radius except for Jupiter only.

The ratio of the resonance point can change by the square root of the radius, because $K = \dfrac{r_1 v_1}{R \cdot M} = \dfrac{r_1}{R \cdot M}\sqrt{\dfrac{GR \cdot M}{r_1}} = \sqrt{\dfrac{r_1 G}{R \cdot M}}$. Therefore we get the next table 2.

Table 2

Square root of ratio of the radius for the planet.

	Earth	Jupiter	Saturn	Uranus	Neptune	Pluto
Mass[10^24kg]	5.977	1899	568.8	86.67	103	0.012
Expected **r1**[m]	**4.28E+05**	**1.36E+08**	**4.07E+07**	**6.20E+06**	**7.37E+06**	**8.59E+02**
Equatorial radius[m]	6.38E+06	7.15E+07	6.03E+07	2.56E+07	2.48E+07	1.14E+06
Stationary orbit[m]	4.22E+07	1.59E+08	1.09E+08	8.47E+07	9.37E+07	1.84E+07
Proximity radius **r0**[m]	**3.84E+08** (Moon)	**1.28E+08** (Metis～)	**1.34E+08** (Pan～)	**4.98E+07** (Cordelia～)	**4.82E+07** (Naiad～)	**1.96E+08** (Charon～)
$\sqrt{r0/r1}$	**29.97**	**0.97-14.92**	**1.81-24.54**	**2.83-58.06**	**2.56(?)-81.03**	150.94-274.61

4. Conclusion

The gravity force is very similar to the electronic force but has 3 times power. The planet arrangement is similar to the electron arrangement but is little influenced by its inner planet.

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