A Relativistic System of Unit
and
the Maxwell Equation*

Yoshio TAKEMOTO**, Seishu SHIMAMOTO***

Department of Mechanical and Electrical Engineering, School of Engineering,
Nippon Bunri University

日 本 文 理 大 学 紀 要
第42巻 第1号
平成26年 3月

(Bulletin of Nippon Bunri University
Vol. 42, No. 1 (2014-MARCH.))

A Relativistic System of Unit and the Maxwell Equation*

Yoshio TAKEMOTO**, Seishu SHIMAMOTO***

Department of Mechanical and Electrical Engineering, School of Engineering,
Nippon Bunri University

Abstract

This paper proposes Coulomb's law of two en bloc electric charges for matrix style. This formula is contained in the Existing laws or the new latitudinous relativistically-extended. Additionally, we can get the several formulas from the modified Maxwell equation. This paper describes part of it in detail.

1. Introduction

There are several systems of unit in the electromagnetic field. We attempt to see the systems as relativistic technique. For that purpose, we need to be well versed in the words, "matrix vector" , "relativistic form" and the Maxwell equation[1]-[3].

1.1. The electrostatic system

Coulomb's law of the electric charge is

$$F = k_1 \frac{Q_1 Q_2}{r^2} \text{ (The force between two electric charges, Coulomb).} \tag{1}$$

Furthermore, the division of formula (1) into two formulas is expressed as:

$$F = k_{1(2)} Q_2 E \text{ (The force in the electric field), } E = k_{1(1)} \frac{Q_1}{r^2} \text{ (The electric field).} \tag{2}$$

Where, $k_{1(1)}$ is the coefficient of a source, $k_{1(2)}$ is the coefficient of an object substance.

*平成25年11月20日受理
**日本文理大学工学部機械電気工学科　教授
***日本文理大学工学部機械電気工学科　准教授

1.2. The electric current system

Coulomb's law of the parallel current is

$F = k_2 \dfrac{I_1 I_2 \mathrm{d}s_1 \mathrm{d}s_2}{r^2}$ (The force between two parallel currents, Ampere・Biot-Savart). (3)

And this is divided into two formulas as:

$F = k_{2(2)} I_2 \mathrm{d}s_2 \mathrm{d}B$ (The force in the magnetic field),

$$\mathrm{d}B = k_{2(1)} \frac{I_1 \mathrm{d}s_1}{r^2} \quad \text{(The magnetic field).} \tag{4}$$

Thus the relation between the current and the charge $I = \dfrac{\mathrm{d}Q}{\mathrm{d}t}$.

Where, $k_{2(1)}$ is the coefficient of a source, $k_{2(2)}$ is the coefficient of an object substance.

1.3. The magnetic system

Coulomb's law of the magnetic charge is $F = k_3 \dfrac{m_1 m_2}{r^2}$ (Gauss). (5)

The division of two formulas is shown as follows:

The force in the magnetic field is $F = k_{3(2)} m_2 B$.

The magnetic field is $B = k_{3(1)} \dfrac{m_1}{r^2}$. (6)

Where, $k_{3(1)}$ is the coefficient of a source, $k_{3(2)}$ is the coefficient of an object substance.

2. The relativistic system of unit

Coulomb's law of the en bloc electric charge $\vec{\rho}$ [4].

$$\vec{\rho} = {}^{+}\begin{bmatrix} \rho_0\gamma & \\ & -\rho_0\gamma\boldsymbol{\beta} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \rho & \\ & -\dfrac{\mathbf{J}_s}{\mathrm{c}} \end{bmatrix}^{+} \tag{7}$$

where $\rho = \dfrac{\rho_0}{\sqrt{1-(\frac{\mathbf{v}}{\mathrm{c}})^2}} = \rho_0\gamma$ (relativistic charge), $\dfrac{\mathbf{J}_s}{\mathrm{c}} = \rho_0 \dfrac{\frac{\mathbf{v}}{\mathrm{c}}}{\sqrt{1-(\frac{\mathbf{v}}{\mathrm{c}})^2}} = \rho_0\gamma\boldsymbol{\beta}$. The state of the stream charge $\mathbf{J}_s$ is similarly local current. The stream charge is the same stationary current for back building. Therefore,

$$\mathbf{J}_s = \rho_0 \frac{\mathbf{v}}{\sqrt{1-(\frac{\mathbf{v}}{\mathrm{c}})^2}} = \rho_0\gamma\mathbf{v} = \rho\mathbf{v} \quad \text{and} \quad \frac{\frac{\mathbf{J}_s}{\mathrm{c}}}{\rho} = \frac{\mathbf{v}}{\mathrm{c}} = \tanh\Theta. \tag{8}$$

The potential of the stationary (or independent of time) en bloc electric charge $\vec{\rho}$ is

$$ {}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\mathbf{A} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \dfrac{\rho_0\gamma_1}{r} & \\ & -\dfrac{\rho_0\gamma_1\boldsymbol{\beta}_1}{r} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \dfrac{\rho_1}{r} & \\ & -\dfrac{\mathbf{J}_{s1}}{\mathrm{c}r} \end{bmatrix}^{+}. \tag{9}$$

And the electromagnetic field is

$$ {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}\mathrm{c}\mathbf{B} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial \mathrm{c}t} & \\ & -\dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\mathbf{A} \end{bmatrix}^{+} $$

$$= {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & -\dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \dfrac{\rho_1}{r} & \\ & -\dfrac{\mathbf{J}_{s1}}{cr} \end{bmatrix}^{+}$$

$$= {}^{-}\begin{bmatrix} \mathrm{div}\dfrac{\mathbf{J}_{s1}}{cr} & \\ & -\mathbf{grad}\dfrac{\rho_1}{r} - \mathbf{irot}\dfrac{\mathbf{J}_{s1}}{cr} \end{bmatrix}^{+}$$

$$= {}^{-}\begin{bmatrix} -\dfrac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3} & \\ & \dfrac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\dfrac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3} \end{bmatrix}^{+} . \tag{10}$$

We define the 4-dimensional Coulomb-Lorentz force of two en bloc electric charges as:

$${}^{-}\begin{bmatrix} f_t & \\ & f \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_0\gamma_2 & \\ & \rho_0\gamma_2\boldsymbol{\beta}_2 \end{bmatrix}^{-}$$

$$= {}^{-}\begin{bmatrix} -\dfrac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3} & \\ & \dfrac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\dfrac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_2 & \\ & \dfrac{\mathbf{J}_{s2}}{c} \end{bmatrix}^{-}$$

$$= {}^{-}\begin{bmatrix} 0 & \\ & \dfrac{\mathbf{r}}{r^3} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \rho_1 & \\ & -\dfrac{\mathbf{J}_{s1}}{c} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_2 & \\ & \dfrac{\mathbf{J}_{s2}}{c} \end{bmatrix}^{-}$$

$$= \left({}^{-}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & -\dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \dfrac{\rho_1}{r} & \\ & -\dfrac{\mathbf{J}_{s1}}{cr} \end{bmatrix}^{+} \right) {}^{-}\begin{bmatrix} \rho_2 & \\ & \dfrac{\mathbf{J}_{s2}}{c} \end{bmatrix}^{-} . \tag{11}$$

This formula (11) means that the 4-dimensional force in the electromagnetic field is divided into two formulas. Therefore we get the following formulas (12), (13) and Theorem1.

Theorem 1. The 4-dimensional Coulomb-Lorentz force of en bloc electric charge

The 4-dimensional force is divided into two formulas as follows:

$${}^{-}\begin{bmatrix} f_t & \\ & f \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_2 & \\ & \dfrac{\mathbf{J}_{s2}}{c} \end{bmatrix}^{-} \text{(The force in the field)} \tag{12}$$

and

$${}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} = {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & -\dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \dfrac{\rho_1}{r} & \\ & -\dfrac{\mathbf{J}_{s1}}{cr} \end{bmatrix}^{+} \text{(The electromagnetic field).} \tag{13}$$

Moreover, we calculate the formula (12) in this Theorem 1. The 4-dimensional force of two en bloc electric charges is

$${}^{-}\begin{bmatrix} f_t & \\ & f \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_0\gamma_2 & \\ & \rho_0\gamma_2\boldsymbol{\beta}_2 \end{bmatrix}^{-} .$$

$$= {}^{-}\begin{bmatrix} \begin{array}{l} -\dfrac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3}\cdot\rho_2 \\ +(\dfrac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\dfrac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3})\bullet\dfrac{\mathbf{J}_{s2}}{c} \end{array} & \\ & \begin{array}{l} -\dfrac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3}\dfrac{\mathbf{J}_{s2}}{c} \\ +(\dfrac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\dfrac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3})\rho_2 \\ -\mathrm{i}(\dfrac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\dfrac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3})\times\dfrac{\mathbf{J}_{s2}}{c} \end{array} \end{bmatrix}^{-} . \tag{14}$$

Therefore,

$$f_t = -\frac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3}\cdot\rho_2 + \left(\frac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\frac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3}\right)\bullet\frac{\mathbf{J}_{s2}}{c} \tag{15}$$

$$\boldsymbol{f} = -\frac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3}\frac{\mathbf{J}_{s2}}{c} + \left(\frac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\frac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3}\right)\rho_2 - \mathrm{i}\left(\frac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\frac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3}\right)\times\frac{\mathbf{J}_{s2}}{c}. \tag{16}$$

Theorem 2. The restriction of Coulomb-Lorentz force to the charge and the current

The force between two en bloc electric charges is an extension of the Coulomb force of two electric charges or two parallel currents. The Coulomb coefficient of the parallel currents is not individual.

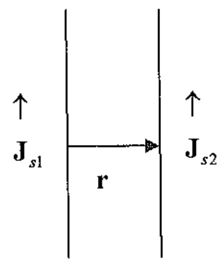

Fig. 1. Schematic diagram of the stream charges.

Fig. 1 shows the schematic diagram of the stream charges in the conductor. $\mathbf{J}_{s1}$ and $\mathbf{J}_{s2}$ are stream charges. Two electric charges or two parallel currents with stream charge lines are parallel.

Proof.

The 4-dimensional Coulomb-Lorentz force of two en bloc electric charges

$${}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} 0 & \\ & \frac{\mathbf{r}}{r^3} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \rho_1 & \\ & -\frac{\mathbf{J}_{s1}}{c} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_2 & \\ & \frac{\mathbf{J}_{s2}}{c} \end{bmatrix}^{-}. \tag{17}$$

Especially the stream charge $\frac{\mathbf{J}_{s1}}{c}, \frac{\mathbf{J}_{s2}}{c}$ is the parallel

$$f_t = -\frac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3}\cdot\rho_2 + \left(\frac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\frac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3}\right)\bullet\frac{\mathbf{J}_{s2}}{c} = 0 \tag{18}$$

$$\boldsymbol{f} = -\frac{\mathbf{J}_{s1}\bullet\mathbf{r}}{cr^3}\frac{\mathbf{J}_{s2}}{c} + \left(\frac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\frac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3}\right)\rho_2 - \mathrm{i}\left(\frac{\rho_1\mathbf{r}}{r^3} - \mathrm{i}\frac{\mathbf{J}_{s1}\times\mathbf{r}}{cr^3}\right)\times\frac{\mathbf{J}_{s2}}{c}$$

$$= -\left(\frac{\mathbf{J}_{s1}}{c}\bullet\frac{\mathbf{J}_{s2}}{c}\right)\frac{\mathbf{r}}{r^3} + \rho_1\rho_2\frac{\mathbf{r}}{r^3} - \mathrm{i}\frac{\mathbf{J}_{s1}\rho_2 - \rho_1\mathbf{J}_{s2}}{c}\times\frac{\mathbf{r}}{r^3}. \tag{19}$$

When the velocity of the charges is zero, the stream charges $\mathbf{J}_{s1} = \mathbf{J}_{s2} = 0$. Thus the charges become ρ_1 and ρ_2, then formula (19) is $\boldsymbol{f} = \rho_1\rho_2\frac{\mathbf{r}}{r^3}$. This formula is Coulomb's law of the electric charge.

The first term $\left(\frac{\mathbf{J}_{s1}}{c}\bullet\frac{\mathbf{J}_{s2}}{c}\right)\frac{\mathbf{r}}{r^3}$ and complex term $\mathrm{i}\frac{\mathbf{J}_{s1}\rho_2 - \rho_1\mathbf{J}_{s2}}{c}\times\frac{\mathbf{r}}{r^3}$ are very small in comparison with the second term $\rho_1\rho_2\frac{\mathbf{r}}{r^3}$.

In the conductor, the 4-dimensional Coulomb-Lorentz force for the second term $\rho_1\rho_2\frac{\mathbf{r}}{r^3}$ and complex term $\mathrm{i}\frac{\mathbf{J}_{s1}\rho_2 - \rho_1\mathbf{J}_{s2}}{c}\times\frac{\mathbf{r}}{r^3}$ are almost cancelled. Then the first term $\left(\frac{\mathbf{J}_{s1}}{c}\bullet\frac{\mathbf{J}_{s2}}{c}\right)\frac{\mathbf{r}}{r^3}$ is used[3],[4]. This formula is Coulomb's law of the parallel currents.

In case the Coulomb coefficient of the electric charge is 1, this theorem 2 is verified by that fact that the Coulomb coefficient of the parallel currents is $\frac{1}{c^2}$.

Q.E.D.

3. The electromagnetic formulas

3.1. The 4-dimensional Coulomb-Lorentz force

The 4-dimensional Coulomb-Lorentz force is

$$ {}^{-}\begin{bmatrix} f_t & \\ & \boldsymbol{f} \end{bmatrix}^{-} = {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_0\gamma_2 & \\ & \rho_0\gamma_2\boldsymbol{\beta}_2 \end{bmatrix}^{-} $$

$$ = {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} \rho_0\dfrac{u_0}{c} & \\ & \rho_0\dfrac{\mathbf{u}}{c} \end{bmatrix}^{-}. \tag{20} $$

(i) The time component is $f_t = E_t\cdot\rho_0\dfrac{u_0}{c}+(\mathbf{E}-\mathrm{i}c\mathbf{B})\bullet\rho_0\dfrac{\mathbf{u}}{c}$. Thus, the real part $E_t\rho_0\dfrac{u_0}{c}+\mathbf{E}\bullet\rho_0\dfrac{\mathbf{u}}{c}$ means the variation of energy. The imaginary part $c\mathbf{B}\bullet\rho_0\dfrac{\mathbf{u}}{c}$ means the variation of rotation energy.

(ii) The space component is $\boldsymbol{f} = E_t\cdot\rho_0\dfrac{\mathbf{u}}{c}+(\mathbf{E}-\mathrm{i}c\mathbf{B})\cdot\rho_0\dfrac{u_0}{c}-\mathrm{i}(\mathbf{E}-\mathrm{i}c\mathbf{B})\times\rho_0\dfrac{\mathbf{u}}{c}$, and then the real part $\mathbf{E}\cdot\rho_0\dfrac{u_0}{c}+E_t\cdot\rho_0\dfrac{\mathbf{u}}{c}-c\mathbf{B}\times\rho_0\dfrac{\mathbf{u}}{c}$ means the 4-dimensional force. The imaginary part $c\mathbf{B}\cdot\rho_0\dfrac{u_0}{c}+\mathbf{E}\times\rho_0\dfrac{\mathbf{u}}{c}$ means the force of rotation.

In case that the force of electric field and the force of magnetic field is in an equilibrium state, when f_t is equal to zero, the time component of the real part is $E_t=-\mathbf{E}\bullet\dfrac{\mathbf{v}}{c}$. This solution indicates that the field E_t is generated. On the other hand, the imaginary part is $\mathbf{B}\bullet\mathbf{v}=0$.

Here, the space component of the real part is $\mathbf{E}=-\dfrac{\mathbf{v}}{c}\times c\mathbf{B}-\dfrac{\mathbf{v}}{c}\cdot E_t$. The field E_t is generated.

The space component of the imaginary part is $c\mathbf{B}=-\mathbf{E}\times\dfrac{\mathbf{v}}{c}$.

$$ \boldsymbol{f}=\mathbf{E}\cdot\rho_0\frac{u_0}{c}+E_t\cdot\rho_0\frac{\mathbf{u}}{c}-c\mathbf{B}\times\rho_0\frac{\mathbf{u}}{c}(=0). \tag{21} $$

Finally, Lenz's law is $\mathbf{E}=-\dfrac{\mathbf{v}}{c}\times c\mathbf{B}-\dfrac{\mathbf{v}}{c}\cdot E_t$.

3.2. The 4-dimensional Gauss's law

The 4-dimensional Gauss's law (derivative type) is

$$ {}^{+}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & \dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} = 4\pi\, {}^{+}\begin{bmatrix} \rho & \\ & \dfrac{\mathbf{J}_s}{c} \end{bmatrix}^{+}. \tag{22} $$

By comparison with the time component

$$ \frac{\partial E_t}{\partial ct}+\mathrm{div}(\mathbf{E}-ic\mathbf{B})=4\pi\rho. $$

And the real part $\mathrm{Div}(E_t,\mathbf{E})=\mathrm{div}\mathbf{E}+\dfrac{\partial E_t}{\partial ct}=4\pi\rho$ is extended to Gauss's law.

The imaginary part means $\mathrm{div}c\mathbf{B}=0$.

By comparison with the space component

$$ \mathbf{grad}\,E_t-\frac{\partial(\mathbf{E}-\mathrm{i}c\mathbf{B})}{\partial ct}-\mathrm{i}\,\mathbf{rot}(\mathbf{E}-\mathrm{i}c\mathbf{B})=4\pi\frac{\mathbf{J}_s}{c}. $$

And the real part $\mathbf{rot}\,c\mathbf{B}-\dfrac{\partial\mathbf{E}}{\partial ct}+\mathbf{grad}\,E_t=4\pi\dfrac{\mathbf{J}_s}{c}$ is Ampere's law.

The imaginary part $\mathbf{rot}\,\mathbf{E}+\dfrac{\partial c\mathbf{B}}{\partial ct}=0$ is Faraday's law of induction.

We integrate the 4-dimensional Gauss's law, and get the integral type[1].

$$ \int_{\partial U} {}^{+}\begin{bmatrix} \mathrm{d}V & \\ & -\mathrm{d}ct\wedge d\mathbf{S} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} $$

$$ = \int_U {}^{+}\begin{bmatrix} \dfrac{\partial}{\partial ct} & \\ & \dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E}-\mathrm{i}c\mathbf{B} \end{bmatrix}^{+} \mathrm{d}ct\wedge\mathrm{d}V $$

$$= 4\pi \int_U {}^{+}\begin{bmatrix} \dfrac{\rho}{r} & \\ & -\dfrac{\mathbf{J}_s}{\mathrm{c}r} \end{bmatrix}^{+} \mathrm{d}\mathrm{c}t \wedge \mathrm{d}V . \tag{23}$$

Where U :4-dimensional domain , ∂U : border of U ,

$\mathrm{d}V = \mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z$, $\mathrm{d}\mathrm{c}t \wedge d\mathbf{S} = (\mathrm{d}\mathrm{c}t \wedge \mathrm{d}S_x)\mathbf{i} + (\mathrm{d}\mathrm{c}t \wedge \mathrm{d}S_y)\mathbf{j} + (\mathrm{d}\mathrm{c}t \wedge \mathrm{d}S_z)\mathbf{k}$

$\mathrm{d}S_x = \mathrm{d}y \wedge \mathrm{d}z$, $\mathrm{d}S_y = \mathrm{d}z \wedge \mathrm{d}x$, $\mathrm{d}S_z = \mathrm{d}x \wedge \mathrm{d}y$ is a 4-dimensional surface element, and

$\mathrm{d}\mathrm{c}t \wedge \mathrm{d}V = \mathrm{d}\mathrm{c}t \wedge \mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z$ is a 4-dimensional volume element.

3.3 The wave equation of the potential and the electromagnetic field

The wave equation of the potential and the electromagnetic field is

$${}^{+}\begin{bmatrix} \dfrac{\partial^2}{\partial \mathrm{c}t^2} - \dfrac{\partial^2}{\partial \mathbf{r}^2} & \\ & 0 \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \phi & \\ & -\mathrm{c}\mathbf{A} \end{bmatrix}^{+} = {}^{+}\begin{bmatrix} \dfrac{\partial}{\partial \mathrm{c}t} & \\ & \dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{+} {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E} - \mathrm{i}\mathrm{c}\mathbf{B} \end{bmatrix}^{+} = 4\pi\, {}^{+}\begin{bmatrix} \rho & \\ & -\dfrac{\mathbf{J}_s}{\mathrm{c}} \end{bmatrix}^{+} \tag{24}$$

$${}^{-}\begin{bmatrix} \dfrac{\partial^2}{\partial \mathrm{c}t^2} - \dfrac{\partial^2}{\partial \mathbf{r}^2} & \\ & 0 \end{bmatrix}^{+} {}^{-}\begin{bmatrix} E_t & \\ & \mathbf{E} - \mathrm{i}\mathrm{c}\mathbf{B} \end{bmatrix}^{+} = 4\pi\, {}^{-}\begin{bmatrix} \dfrac{\partial}{\partial \mathrm{c}t} & \\ & -\dfrac{\partial}{\partial \mathbf{r}} \end{bmatrix}^{-} {}^{+}\begin{bmatrix} \rho & \\ & -\dfrac{\mathbf{J}_s}{\mathrm{c}} \end{bmatrix}^{+}$$

$$= 4\pi\, {}^{-}\begin{bmatrix} \dfrac{\partial \rho}{\partial \mathrm{c}t} + \mathrm{div}\dfrac{\mathbf{J}_s}{\mathrm{c}} & \\ & -\dfrac{\partial \frac{\mathbf{J}_s}{\mathrm{c}}}{\partial \mathrm{c}t} - \mathbf{grad}\,\rho - \mathrm{i}\,\mathbf{rot}\dfrac{\mathbf{J}_s}{\mathrm{c}} \end{bmatrix}^{+} \tag{25}$$

(A numerical expression is as follows: $\dfrac{\partial^2}{\partial \mathrm{c}t^2}$ is equal to $\dfrac{\partial^2}{\mathrm{c}^2 \partial t^2}$)

The time component is $\mathrm{Div}(\rho, \dfrac{\mathbf{J}_s}{\mathrm{c}}) = \dfrac{\partial \rho}{\partial \mathrm{c}t} + \mathrm{div}\dfrac{\mathbf{J}_s}{\mathrm{c}} = 0$.

The space component is $\mathbf{Rot}\,(\rho, \dfrac{\mathbf{J}_s}{\mathrm{c}}) = -\dfrac{\partial \frac{\mathbf{J}_s}{\mathrm{c}}}{\partial \mathrm{c}t} - \mathbf{grad}\,\rho - \mathrm{i}\,\mathbf{rot}\dfrac{\mathbf{J}_s}{\mathrm{c}} = 0$.

Thus the real part $\dfrac{\partial \frac{\mathbf{J}_s}{\mathrm{c}}}{\partial \mathrm{c}t} + \mathbf{grad}\,\rho = 0$ means the 4-dimensional force, The imaginary part $\mathbf{rot}\,\mathbf{J}_s = 0$ means the force of rotation.

4. Conclusion

The electric charge and the electric current are two states of "en bloc". When we see the two objects from this point of view, it is not much to say that two Coulomb's law of the electric charge and the parallel current are two sides of one and the same law.

We considered that "Time" and "Space" are equivalent. Therefore the velocity of light "c" which appears in the Maxwell equation is a factor of proportionality of the distance and the time that we decided artificially.

References

[1] Y. Takemoto, Vector Analysis on Time-Space, Bull. of NBU, Vol. 24, No.1 (1996-Feb.) pp. 75-89.

[2] Y. Takemoto, New Notation and Relativistic Form of the 4-dimensional Vector in Time-Space, Bull. of NBU, Vol. 34, No.1 (2006-Mar.) pp. 32-38.

[3] Y. Takemoto, A New Form of Equation of Motion for a Moving Charge and the Lagrangian, Bull. of NBU, Vol. 35, No.1 (2007-Mar.) pp. 1-9.

[4] Y. Takemoto, S. Shimamoto, The Basic and New concept of the Lorentz transformation in a Minkowski Space, Bull. of NBU, Vol. 40, No.2 (2012-Oct.) pp.1-10.