~~電磁気的重力理論(一般相対性理論との比較)~~

The Electromagnetic Gravitational theory
(-Comparison with General theory of relativity-)

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This Electromagnetic Gravitational theory is a Gravitational theory which is imitating the Electromagnetic theory.

Its Gravitation is induced from the Electromagnetic field containing "time component" which is new item, not Gravitation induced by General theory of relativity in the weak field.

In this paper, we discuss and calculate "the system of equations" which appeared in above Electromagnetic Gravitational theory.

This system of equations is correspond to the system of General theory of relativity, and we get the the almost same values of the advance of perihelion and the shift of light which are caused by gravitational field.

This means that the principle of electromagnetic with "time component" is worthy of comparison to the principle of curved spacetime.

Our image in this paper is under Anti de-Sitter space. But this image is not so particular about the discussion and calculations below.

Contents:

In §1 for preliminaries we mention the equation of motion of Newton type.

In §2 we discuss the Newton and the Kepler type's system of equations.

In §3 we discuss and calculate about the typical case.

§1. Preliminaries.

We put $M_G = \dfrac{GM}{c^2}$ as the gravitational constant, and then we have a Newton type's system of equations which is induced from the electromagnetic gravitational force.[2)]

$$(1)_{ct}\quad \frac{d^2ct}{d\tau^2} = -\frac{M_G}{r^2}\left(\frac{\mathbf{r}}{r}\cdot\frac{d\mathbf{r}}{d\tau}\right)\frac{dct}{d\tau}$$

$$(2)_{r,\theta,\phi}\quad \frac{d^2\mathbf{r}}{d\tau^2} = -\frac{M_G}{r^2}\frac{\mathbf{r}}{r}\left(\frac{dct}{d\tau}\right)^2 + i\frac{M_G}{r^2}\left(\frac{\mathbf{r}}{r}\times\frac{d\mathbf{r}}{d\tau}\right)\frac{dct}{d\tau}$$

And the metric is the Minkowski's metric $ds^2 = -dct^2 + d\mathbf{r}^2 = -dct^2 + dx^2 + dy^2 + dz^2$.

We can rewrite this coordinate (x, y, z) by the spherical polar coordinate (r, θ, ϕ), that is, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$. Then we get the system of equations of Newton type.

Theorem 1 (The equations of motion at the spherical polar coordinate.)

The metric is $ds^2 = -dct^2 + dr^2 + r^2(\sin^2\theta d\phi^2 + d\theta^2)$ and the system of equations is

$$(1)_{ct}\quad \frac{d^2ct}{d\tau^2} = -\frac{M_G}{r^2}\frac{dr}{d\tau}\frac{dct}{d\tau}\cdots \text{ (the direction of time)},$$

$$(2)_{r}\quad \frac{d^2r}{d\tau^2} = -\frac{M_G}{r^2}\left(\frac{dct}{d\tau}\right)^2 + \frac{1}{\underline{r}}\left\{\left(r\frac{d\theta}{d\tau}\right)^2 + \left(r\sin\theta\frac{d\phi}{d\tau}\right)^2\right\}\cdots \text{ (the direction of radius)},$$

$$(3)_{\theta}\quad \frac{d}{d\tau}\left(r\frac{d\theta}{d\tau}\right) = \underline{-i\frac{M_G}{r^2}\left(r\sin\theta\frac{d\phi}{d\tau}\right)\frac{dct}{d\tau}} - \frac{1}{r}\left(\frac{dr}{d\tau}\right)\left(r\frac{d\theta}{d\tau}\right) - \frac{\cot\theta}{r}\left(r\sin\theta\frac{d\phi}{d\tau}\right)^2\}$$
$$\cdots \text{ (the direction of longitude)},$$

$$(4)_{\phi}\quad \frac{d}{d\tau}\left(r\sin\theta\frac{d\phi}{d\tau}\right) = \underline{i\frac{M_G}{r^2}\left(r\frac{d\theta}{d\tau}\right)\frac{dct}{d\tau}} - \frac{1}{r}\left(\frac{dr}{d\tau}\right)\left(r\sin\theta\frac{d\phi}{d\tau}\right) + \frac{\cot\theta}{r}\left(r\frac{d\theta}{d\tau}\right)\left(r\sin\theta\frac{d\phi}{d\tau}\right)\}$$
$$\cdots \text{ (the direction of latitude)}.$$

where underlined parts are complex terms.

We expect that all informations of our main purposes are included in this system of equations.

§2. The equations of motion of Newton's type.

We consider the two-body problem concerned with the sun and the planet. Then the planet moves on the equator of the sun. Therefore, we put $\theta = \frac{\pi}{2} - i\Omega$, where Ω is a parameter relating to the angle of rotation on the orbit.

Then we change the imaginary parts to the real and get a real coefficient equation as follows;

The metric is $ds^2 = -dct^2 + dr^2 + r^2(\cosh^2\Omega d\phi^2 - d\Omega^2)$ and the system of equations is

$$(1)\quad \frac{d^2ct}{d\tau^2} = -\frac{M_G}{r^2}\frac{dr}{d\tau}\frac{dct}{d\tau}\cdots \text{ (the direction of time)},$$

(2) $\dfrac{d^2r}{d\tau^2} = -\dfrac{M_G}{r^2}(\dfrac{dct}{d\tau})^2 + \dfrac{1}{r}\{(r\cosh\Omega\dfrac{d\phi}{d\tau})^2 - (r\dfrac{d\Omega}{d\tau})^2\}\cdots$ (the direction of radius),

(3) $\dfrac{d}{d\tau}(r^2\dfrac{d\Omega}{d\tau}) = (\dfrac{M_G}{r^2}\dfrac{dct}{d\tau} - \sinh\Omega\dfrac{d\phi}{d\tau})(r^2\cosh\Omega\dfrac{d\phi}{d\tau})\cdots$ (the longitude areal velocity),

(4) $\dfrac{d}{d\tau}(r^2\cosh\Omega\dfrac{d\phi}{d\tau}) = (\dfrac{M_G}{r^2}\dfrac{dct}{d\tau} - \sinh\Omega\dfrac{d\phi}{d\tau})(r^2\dfrac{d\Omega}{d\tau})\cdots$ (the latitude areal velocity).

For doing the good discussion, we translate the above equations of Newton type to the equations of Kepler type.

Theorem. (The system of equations of Kepler's type.)

The system of equations is

(1)' $\dfrac{dct}{d\tau} = C_0 e^{\frac{M_G}{r}}$ •••••• (the kinetic energy),

(2)' $\dfrac{d^2}{d\tau^2}(r\sinh\Omega) = -(\dfrac{M_G}{r^2}\dfrac{dct}{d\tau})(\tanh\Omega - r\cosh\Omega\dfrac{d\phi}{dct})\cosh\Omega(\dfrac{dct}{d\tau})$

•••••• (the structure of space),

(3)' $(r^2\dfrac{d\Phi}{d\tau})^2 = (r^2\cosh\Omega\dfrac{d\phi}{d\tau})^2 - (r^2\dfrac{d\Omega}{d\tau})^2 = C^2$ ••• (the law of equal areas),

(4)' $r^2\cosh\Omega\dfrac{d\phi}{d\tau} = C\cosh\Theta'(\geqq 0),\quad r^2\dfrac{d\Omega}{d\tau} = -C\sinh\Theta'$

$$\Theta' = \int(\sinh\Omega\frac{d\phi}{d\tau} - \frac{M_G}{r^2}\frac{dct}{d\tau})d\tau \text{••• (the internal rotation).}$$

All informations in physics are contained in this system of equations.

We put the angular velocity $\dfrac{d\Phi}{d\tau} = \sqrt{(\cosh\Omega\dfrac{d\phi}{d\tau})^2 - (\dfrac{d\Omega}{d\tau})^2}$, the orbit speed $r\dfrac{d\Phi}{d\tau}$

and the main equation $(\dfrac{d\frac{1}{r}}{d\Phi})^2 = (\dfrac{dr}{r^2 d\Phi})^2 = -\dfrac{c^2}{C^2} + \dfrac{C_0^{\,2}}{C^2}e^{2\frac{M_G}{r}} - \dfrac{1}{r^2}$.

We call C_0, C the kinetic energy constant and equal areas constant respectively.

Proof

(i) The kinetic energy.

From the equation $\dfrac{d^2ct}{d\tau^2} = -\dfrac{M_G}{r^2}\dfrac{dr}{d\tau}\dfrac{dct}{d\tau}\cdots(1)$.

$(\dfrac{dct}{d\tau})^{-1}\dfrac{d^2ct}{d\tau^2} = -\dfrac{M_G}{r^2}\dfrac{dr}{d\tau}\ \therefore\ \dfrac{d}{d\tau}\log(\dfrac{dct}{dc\tau}) = \dfrac{d}{d\tau}(\dfrac{M_G}{r})$, Therefore, we get the equation of the kinetic energy $\dfrac{dct}{d\tau}=\dfrac{c}{\sqrt{1-(\dfrac{v}{c})^2}}=C_0e^{\frac{M_G}{r}}\cdots(1)'$(the kinetic energy)、where $e^{\frac{M_G}{r}}$ is a potencial energy in the (Anti de-Sitter)space and in its tangent space usally potential $\dfrac{M_G}{r}$. This image is not so particular about this discussion and calculations below.

This equation (1)’ means the law of the conservation of energy. Because at any two points, we put at two points the heights r_0, r and speeds v_0, v respectively. Then

$$C_0=\frac{c}{\sqrt{1-(\dfrac{v_0}{c})^2}}e^{-\frac{M_G}{r_0}}=\frac{c}{\sqrt{1-(\dfrac{v}{c})^2}}e^{-\frac{M_G}{r}}.\quad \therefore \frac{\sqrt{1-(\dfrac{v_0}{c})^2}}{\sqrt{1-(\dfrac{v}{c})^2}}=e^{-\frac{M_G}{r_0}+\frac{M_G}{r}}.$$

$\therefore -\dfrac{1}{2}(\dfrac{v_0}{c})^2+\dfrac{1}{2}(\dfrac{v}{c})^2+\bullet\bullet\bullet=-\dfrac{M_G}{r_0}+\dfrac{M_G}{r}+\bullet\bullet\bullet$, where $M_G=\dfrac{GM}{c^2}$.

This means the conservation of energy as follows,

$$\frac{1}{2}mv^2+(-G\frac{mM}{r})=\frac{1}{2}mv_0{}^2+(-G\frac{mM}{r_0})=constant.$$

(ii) The law of equal areas.

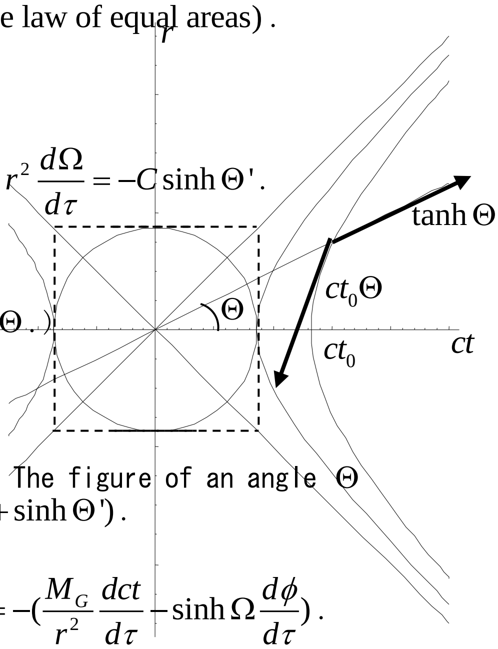
From the equation $(3)\times(r^2\dfrac{d\Omega}{d\tau})-(4)\times(r^2\cosh\Omega\dfrac{d\phi}{d\tau})$, we get

$$(r^2\frac{d\Phi}{d\tau})^2=(r^2\cosh\Omega\frac{d\phi}{d\tau})^2-(r^2\frac{d\Omega}{d\tau})^2=C^2\cdots\cdots(3)'\ \text{(the law of equal areas)}.$$

(iii) The internal rotation.

From$(3)'$, we can put $r^2\cosh\Omega\dfrac{d\phi}{d\tau}=C\cosh\Theta'(\geqq 0)$、$r^2\dfrac{d\Omega}{d\tau}=-C\sinh\Theta'$.

Then $\dfrac{r^2\dfrac{d\Omega}{d\tau}}{r^2\cosh\Omega\dfrac{d\phi}{d\tau}}=-\tanh\Theta'$ holds. (Cf. $\dfrac{dr}{dct}=\tanh\Theta$.)



The figure of an angle Θ

Therefore, from the equation $(4)-(3)$, we get

$$\frac{d}{d\tau}(\sinh\Theta'+\cosh\Theta')=-(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau})(\cosh\Theta'+\sinh\Theta').$$

$$\therefore \frac{de^{\Theta'}}{d\tau}=-(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau})e^{\Theta'}\ \therefore \frac{d\Theta'}{d\tau}=\frac{d}{d\tau}\log e^{\Theta'}=-(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau}).$$

$$\therefore \Theta'=-\int(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau})d\tau\cdots\cdots(4)'\text{(the internal rotation)}.$$

(iv) The structure of space and the Minkowski's metric.

From the equations $(3)'$ and $(2)\times 2(\frac{dr}{d\tau})-(1)\times 2(\frac{dct}{d\tau})$, we get

$$\frac{d}{d\tau}(\frac{dct}{d\tau})^2-\frac{d}{d\tau}(\frac{dr}{d\tau})^2-\frac{d}{d\tau}(\frac{C^2}{r^2})=0.$$

This means the Minkouski's metric in the time-space. Therefore,

$(\frac{dct}{d\tau})^2-(\frac{dr}{d\tau})^2-(\frac{C^2}{r^2})=c^2$ holds. And then $(\frac{dr}{d\tau})^2=-c^2+C_0{}^2e^{2\frac{M_G}{r}}-\frac{C^2}{r^2}$.

By this equation and the law of equal area $r^2\frac{d\Phi}{d\tau}=C$ (3)', we get the main equation

$(\frac{d\frac{1}{r}}{d\Phi})^2=(\frac{dr}{r^2d\Phi})^2=-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}$ in the tangent space.

And more by the $(2)\times\sinh\Omega+(3)\times\frac{1}{r}\cosh\Omega$,

$$\frac{d^2r}{d\tau^2}\sinh\Omega+\frac{d}{d\tau}(r\frac{d\Omega}{d\tau})\cosh\Omega+\frac{dr}{d\tau}(\frac{d\Omega}{d\tau})\cosh\Omega$$

$$=-(\frac{M_G}{r^2}\frac{dct}{d\tau})(\tanh\Omega-r\cosh\Omega\frac{d\phi}{dct})\cosh\Omega(\frac{dct}{d\tau})-\frac{1}{r}(r\frac{d\Omega}{d\tau})^2\sinh\Omega \quad \text{holds.}$$

Therefore, we get

$$\frac{d^2}{d\tau^2}(r\sinh\Omega)=\frac{d}{d\tau}\{\frac{d}{d\tau}(r\sinh\Omega)\}=\frac{d}{d\tau}\{\frac{dr}{d\tau}\sinh\Omega+r\cosh\Omega\frac{d\Omega}{d\tau}\},$$

$$=\frac{d^2r}{d\tau^2}\sinh\Omega+\frac{dr}{d\tau}\cosh\Omega\frac{d\Omega}{d\tau}+\sinh\Omega\frac{d\Omega}{d\tau}(r\frac{d\Omega}{d\tau})+\frac{d}{d\tau}(r\frac{d\Omega}{d\tau})\cosh\Omega,$$

$$=-(\frac{M_G}{r^2}\frac{dct}{d\tau})(\tanh\Omega-r\cosh\Omega\frac{d\phi}{dct})\cosh\Omega(\frac{dct}{d\tau})\cdots(2)'\ \text{(the structure of space)}.$$

(A consideration in special case)

This equation indicate a motion of free oscillation by Hooke's law as follows,

$$\frac{d^2}{d\tau^2}(\underline{\underline{r\sinh\Omega}})=-\frac{M_G}{r^3}(\frac{dct}{d\tau})^2(\underline{\underline{r\sinh\Omega}}-r^2\cosh\Omega\frac{d\phi}{dct}\cdot\cosh\Omega).$$

When this oscillation is stable, the idebtity $\tanh\Omega\equiv r\cosh\Omega\frac{d\phi}{dct}$ holds, and this formula decide the value of parameter Ω of the angle of rotation on the orbit.

From this idebtity and the relation $\frac{d}{d\tau}(r\sinh\Omega)=\frac{dr}{d\tau}\sinh\Omega+r\cosh\Omega\frac{d\Omega}{d\tau}$, The relations $\dfrac{d^2}{d\tau^2}(r\sinh\Omega)\equiv 0$、$\dfrac{d}{d\tau}(r\sinh\Omega)\equiv 0$ and

$\dfrac{dr}{dct}\cdot r\cosh\Omega\dfrac{d\phi}{dct}\equiv -r\dfrac{d\Omega}{dct}$ holds.

Last equation yield two important equations, that is,

(i) Two parameters Θ, $\Theta'$are equivalent, because $\dfrac{dr}{dct}\equiv -\dfrac{r\dfrac{d\Omega}{dct}}{r\cosh\Omega\dfrac{d\phi}{dct}}$.

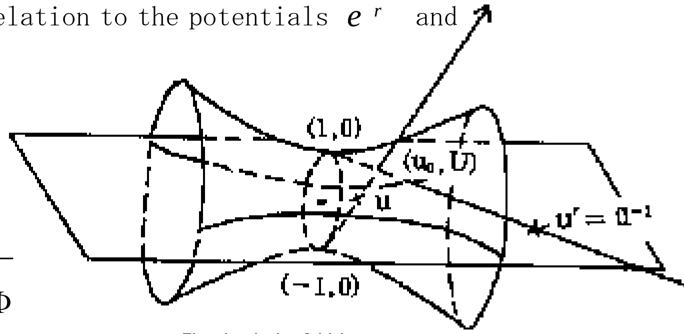
(ii) the composition of energies represent not simply the sum but the product of two energies, because we put $\tanh\Omega_0 = r\cosh\Omega\dfrac{d\phi}{dct}$, then we get

$$\frac{dt}{d\tau}=\frac{1}{\sqrt{1-(\frac{dr}{dct})^2-r^2\{(\cosh\Omega\frac{d\phi}{dct})^2-(\frac{d\Omega}{dct})^2\}}},$$

$$=\frac{1}{\sqrt{1-(\frac{dr}{dct})^2}}\cdot\frac{1}{\sqrt{1-(r\cosh\Omega\frac{d\phi}{dct})^2}}(=\cosh\Theta\cdot\cosh\Omega_0).$$

§3. Calculation of Newton type's equations of motion.

We differentiate the kinetic energy (1) $\dfrac{dct}{d\tau}=C_0 e^{\frac{M_G}{r}}$ by the orbit angle Φ, $d\Phi=\sqrt{(\cosh\Omega d\phi)^2-(d\Omega)^2}$. Then we get the relation to the potentials $e^{\frac{M_G}{r}}$ and $\dfrac{M_G}{r}$ of space and its tangent space as follows,



The Anti de-Sitter spaxe

$$\frac{d}{d\Phi}(\frac{dct}{d\tau})=\frac{d}{d\Phi}(C_0e^{\frac{M_G}{r}})=C_0\frac{e^{\frac{M_G}{r}}d\frac{M_G}{r}}{d\Phi}=C_0\frac{d\frac{M_G}{r}}{e^{-\frac{M_G}{r}}d\Phi}$$

Therefore, we get the main quation in the Anti de-Sitter space as follows,

$$(\frac{de^{\frac{M_G}{r}}}{d\Phi})^2=(\frac{M_Gd\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi})^2=M_G{}^2(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2})e^{2\frac{M_G}{r}}\quad\cdot\cdot(*).$$

The main equation in the above §2 theorem is the one of the potential relation to the tangent space of it.

We differentiate this main equation (*), then

$$2\left(\frac{M_G d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)\frac{d}{d\Phi}\left(\frac{M_G d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)$$

$$=M_G{}^2\left[\left(2M_G\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{2}{r}\right)e^{2\frac{M_G}{r}}+\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\right)2M_G e^{2\frac{M_G}{r}}\right]\left(\frac{d\frac{1}{r}}{d\Phi}\right).$$

$$\therefore \frac{d}{e^{-\frac{M_G}{r}}d\Phi}\left(\frac{d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)=\left(M_G\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r}\right)e^{2\frac{M_G}{r}}+\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\right)M_G e^{2\frac{M_G}{r}},$$

$$=-\left(M_G\frac{c^2}{C^2}+\frac{1}{r}+M_G\frac{1}{r^2}\right)e^{2\frac{M_G}{r}}+2M_G\frac{C_0{}^2}{C^2}e^{4\frac{M_G}{r}}\text{、}\ e^{2\frac{M_G}{r}}=1+2\frac{M_G}{r}+2\frac{M_G{}^2}{r^2}+\cdots,$$

$$\fallingdotseq\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)M_G+\left\{\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2-1\right\}\frac{1}{r}.$$

Therefore, we get the following differential equation,

$$\frac{d^2\frac{1}{r}}{\left(e^{-\frac{M_G}{r}}d\Phi\right)^2}=-\left\{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2\right\}\left[\frac{1}{r}-\frac{\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)M_G}{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}\right].$$

This solution is an ellipse and contains a circle, a parabola, a hyperbola.

$$\frac{1}{r}=\frac{\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)M_G}{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}\left(1+e\cos\left(\sqrt{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}\cdot e^{-\frac{M_G}{r}}\Phi\right)\right).$$

Where e is a constant of integration i.e. an eccentricity.

(Cf1.) The Minkouski's metric corresponds to
the Schwarzschild metric $ds^2 = -d'ct^2 + d'r^2 + r^2(\sin^2\theta d\phi^2 + d\theta^2)$
by the $d'ct = \sqrt{1-\frac{2M_G}{r}}dct$ and the $d'r = \frac{1}{\sqrt{1-\frac{2M_G}{r}}}dr$.

And the main equation in General theory of relativity is

$(\frac{1}{\sqrt{1-\frac{2M_G}{r}}}\frac{d\frac{1}{r}}{d\phi})^2 = -\frac{c^2}{C^2} + \frac{{C_0}^2}{C^2}\frac{1}{1-\frac{2M_G}{r}} - \frac{1}{r^2}$. And this is similar to the main equation

(*) therefore we can expect the same result below, that is, Example 1 and Example 2.

Example 1 (*The perihelion precession of Mercury*)

We calculate the period of the Mercury orbit with respect to the angle $\varphi = e^{-\frac{M_G}{r}}\Phi$ as follows; when the Mercury travel around the sun, the value of an angle φ is

From $\sqrt{1-(8\frac{{C_0}^2}{C^2} - 2\frac{c^2}{C^2}){M_G}^2}\cdot\varphi = 2\pi$, $\varphi = \frac{2\pi}{\sqrt{1-(8\frac{{C_0}^2}{C^2} - 2\frac{c^2}{C^2}){M_G}^2}}$.

Therefore, the advance of perihelion is

$$\frac{2\pi}{\sqrt{1-(8\frac{{C_0}^2}{C^2} - 2\frac{c^2}{C^2}){M_G}^2}} - 2\pi \fallingdotseq 2\pi(\frac{4{C_0}^2 - c^2}{C^2}){M_G}^2.$$

We substitute the concrete value for this formula. as follows;

The light speed is $c = 2.99792458\times10^8\mathrm{m}\cdot\sec^{-1}$.

The Newton's constant of gravity is $G = 6.673\times10^{-11}m^3\cdot kg^{-1}\cdot\sec^{-2}$.

The weight of the sun is $M = 1.989\times10^{30}$ kg.

Therefore, we get $M_G = \frac{GM}{c^2} = 1476.55\mathrm{m}$.

And more, we put r_1 (a perihelion) is 4.60012×10^{10}m,
r_2 (an aphelion) is 6.98169×10^{10}m

This two values are solutions of the equation $(\frac{dr}{d\tau})^2 = -c^2 + {C_0}^2 e^{2\frac{M_G}{r}} - \frac{C^2}{r^2} = 0$.

Therefore, the values C_0 (the kinetic constant) and C (the equal areas constant) satisfy following simultaneous equations.

$$\frac{C^2}{r_1^{\ 2}} - C_0^{\ 2} e^{2\frac{M_G}{r_1}} + c^2 = 0,\ \frac{C^2}{r_2^{\ 2}} - C_0^{\ 2} e^{2\frac{M_G}{r_2}} + c^2 = 0、\ C_0, C > 0.$$

Then we get $C = 2.7129204353541553 \times 10^{15} m^2 \cdot \sec^{-1}$,

C_0=$2.997924541779737 \times 10^8 m \cdot \sec^{-1}$ ($\approx c$： the light speed).

Therefore, the value of the advance of perihelion is

$$360 \times 60(\min) \times 60(\sec) \times (\frac{4C_0^{\ 2} - c^2}{C^2}) M_G^{\ 2} \times 415(\text{round}) = 42.95626382458457''.$$

(Cf2.) In General theory of relativity,

$$2\pi \frac{3c^2}{C^2} M_G^{\ 2} = 2\pi \frac{3M_G}{r_m(1-e^2)}(\text{rad}) = 43.03''(\textit{per } 100 \textit{years}).$$

Example 2 (The shift of light by the strong gravitational field.)

Generally, we put the speed v_1 at the perihelion r_1. then we devide the equal areas constant $r\frac{d\Phi}{d\tau} = \frac{r_1 v_1}{\sqrt{1-(\frac{v_1}{c})^2}} = C$ by the kinetic energy constant

$\frac{dct}{d\tau} = \frac{c}{\sqrt{1-(\frac{v_1}{c})^2}} = C_0 e^{\frac{M_G}{r_1}}$, then we get the ratio $\frac{C}{C_0 e^{\frac{M_G}{r_1}}} = r_1 \frac{v_1}{c}$ of two constants.

We put the radius of the sun $R_\odot = 6.955 \times 10^8 m$ as the perihelion r_1 of light.

And we take the speed v_1 very close to the light speed. Then constant C and constant C_0 diverge to infinity. But the ratio of two constants converge, that is,

when $v_1 \to c(\textit{light speed})$, $\frac{C}{C_0} = r_1 \frac{v_1}{c} e^{\frac{M_G}{r_1}} \to R_\odot e^{\frac{M_G}{R_\odot}} \fallingdotseq R_\odot$.

Therefore, the main equation converge to

$$(\frac{de^{\frac{M_G}{r}}}{d\Phi})^2 = (\frac{M_G d\frac{1}{r}}{e^{-\frac{M_G}{r}} d\Phi})^2 = M_G^{\ 2}(\frac{1}{R_\odot^{\ 2}} e^{2\frac{M_G}{r}} - \frac{1}{r^2}) e^{2\frac{M_G}{r}}$$

And the orbit of the light is the hyperbola, that is,

$$\frac{1}{r}=\frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}\left(1+e\cos\left(\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi\right)\right),\quad \varphi=e^{-\frac{M_G}{r}}\Phi$$

(A) When the light is at the point $R_\odot=r_1$ (a perihelion), that is,

$$\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi=0,\ \text{ then }\ \frac{1}{R_\odot}=\frac{1}{r_1}=\frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}(1+e)$$

Therefore, we get an eccentricity of the hyperbola, that is,

$$e=\frac{1-\frac{8M_G{}^2}{R_\odot{}^2}}{\frac{2M_G}{R_\odot}}-1\fallingdotseq\frac{R_\odot}{2M_G}=2.35515\times10^5,\ \text{ where }\ \frac{M_G}{R_\odot}\ \text{is}\ 2.12301\times10^{-6}.$$

(B) When the light is at the point r_2 (an aphelion), that is,

$$\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi=\pi,\ \text{ then }\ \frac{1}{r_2}=\frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}(1-e)$$

Therefore, we get a value of r_2, that is,

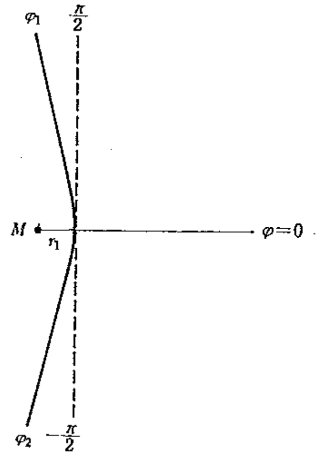
$$r_2=\frac{1-\frac{8M_G{}^2}{R_\odot{}^2}}{\frac{2M_G}{R_\odot{}^2}(1-e)}\fallingdotseq\frac{R_\odot}{2M_G(1-e)}R_\odot=\frac{e}{1-e}R_\odot=-6.95503\times10^8(<-R_\odot)$$

(C) When the light is at the point $r\to\infty$ (an asyanmptote) , that is,

By the use of property $\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi\neq\pi$ and more $\frac{\pi}{2}<\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi<\pi$,

Then $0=(\frac{1}{\infty}=)\frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}\left(1+e\cos\left(\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi\right)\right)$,

$$\therefore\ \cos\left(\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi\right)=-\frac{1}{e}<0\ \therefore\ \sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}\cdot\varphi-\frac{\pi}{2}\fallingdotseq\frac{1}{e}$$



$$\therefore \sqrt{1-\frac{8{M_G}^2}{{R_\odot}^2}}\cdot\varphi \fallingdotseq \pm\left(\frac{\pi}{2}+\frac{1}{e}\right) \therefore \varphi \fallingdotseq \pm\frac{\frac{\pi}{2}+\frac{1}{e}}{\sqrt{1-\frac{8{M_G}^2}{{R_\odot}^2}}}$$

Therefore, we get a shift angle of light, that is,

$$\Delta\varphi = 2\frac{\frac{\pi}{2}+\frac{1}{e}}{\sqrt{1-\frac{8{M_G}^2}{{R_\odot}^2}}} - \pi = \left(\frac{\pi}{\sqrt{1-\frac{8{M_G}^2}{{R_\odot}^2}}} - \pi\right) + \frac{\frac{2}{e}}{\sqrt{1-\frac{8{M_G}^2}{{R_\odot}^2}}} \text{、} \frac{1}{e} \fallingdotseq \frac{2M_G}{R_\odot}$$

$$\fallingdotseq \frac{4M_G}{R_\odot} = 8.49204\times10^{-6}(\mathrm{rad}) = 1.75161''$$

(Cf3.) In General theory of relativity, the Schwarzschild metric of the light is $-d'ct^2 + d'r^2 + r^2(\sin^2\theta d\phi^2 + d\theta^2) = 0$ and

$$4M_G\frac{C_0}{C} = \frac{4M_G}{R_\odot} = \frac{4\times1476.55}{6.955\times10^8} = 8.5\times10^{-6}(\mathrm{rad}) = 1.75''.$$

References

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