The Electromagnetic Gravitational Theory*
(Comparison with General Theory of Relativity)

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Abstract

This electromagnetic gravitational theory is a gravitational theory which is imitates the electromagnetic theory.

The gravitation in this theory is induced from the electromagnetic field containing "time component" which is a new item, not a gravitation induced by general theory of relativity in the weak field.

In this paper, we discuss and calculate "the system of equations" which appeared in the above electromagnetic gravitational theory.

This system of equations corresponds to the system of general theory of relativity, and we get the almost same values of the advance of perihelion and the shift of light which are caused by the strong gravitational field.

This means that the principle of electromagnetics with "time component" is worthy of comparison to the principle of curved space-time.

Our image in this paper is under the anti de-Sitter space. But this image is not so particular about the discussion and calculations below.

Contents:

In § 1 for preliminaries we mention the equation of motion of Newton type.

In § 2 we discuss the Newton and the Kepler type system of equations.

In § 3 we discuss and calculate about a typical case.

§ 1. Preliminaries.

We put $M_G = \frac{GM}{c^2}$ as the gravitational constant, and then we have a system of Newton type equations to which the electromagnetic gravitational force leads, as follows ; [2)]

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$$(1)_{ct}\quad \frac{d^2ct}{d\tau^2}=-\frac{M_G}{r^2}\left(\frac{\mathbf{r}}{r}\cdot\frac{d\mathbf{r}}{d\tau}\right)\frac{dct}{d\tau}$$

$$(2)_{r,\theta,\phi}\quad \frac{d^2\mathbf{r}}{d\tau^2}=-\frac{M_G}{r^2}\frac{\mathbf{r}}{r}\left(\frac{dct}{d\tau}\right)^2+i\frac{M_G}{r^2}\left(\frac{\mathbf{r}}{r}\times\frac{d\mathbf{r}}{d\tau}\right)\frac{dct}{d\tau}$$

And the metric is the Minkowski's metric $ds^2=-dct^2+d\mathbf{r}^2=-dct^2+dx^2+dy^2+dz^2$.

We represent this system of equations by the spherical polar coordinate (r,θ,ϕ), that is,

$$\begin{cases} x=r\sin\theta\cos\phi, \\ y=r\sin\theta\sin\phi, \\ z=r\cos\theta. \end{cases}$$

Then we get the system of equations of Newton type.

Theorem 1 (The equations of motion by the spherical polar coordinate.)[2)]

The metric is $ds^2=-dct^2+dr^2+r^2(\sin^2\theta d\phi^2+d\theta^2)$ and the system of equations is

$$(1)_{ct}\quad \frac{d^2ct}{d\tau^2}=-\frac{M_G}{r^2}\frac{dr}{d\tau}\frac{dct}{d\tau}\cdots\text{(the direction of time),}$$

$$(2)_r\quad \frac{d^2r}{d\tau^2}=-\frac{M_G}{r^2}\left(\frac{dct}{d\tau}\right)^2+\frac{1}{r}\left\{\left(r\frac{d\theta}{d\tau}\right)^2+\left(r\sin\theta\frac{d\phi}{d\tau}\right)^2\right\}\cdots\text{(the direction of radius),}$$

$$(3)_\theta\quad \frac{d}{d\tau}\left(r\frac{d\theta}{d\tau}\right)=\underline{-i\frac{M_G}{r^2}\left(r\sin\theta\frac{d\phi}{d\tau}\right)\frac{dct}{d\tau}}-\frac{1}{r}\left(\frac{dr}{d\tau}\right)\left(r\frac{d\theta}{d\tau}\right)-\frac{\cot\theta}{r}\left(r\sin\theta\frac{d\phi}{d\tau}\right)^2\cdots\text{(the direction of longitude),}$$

$$(4)_\phi\quad \frac{d}{d\tau}\left(r\sin\theta\frac{d\phi}{d\tau}\right)=\underline{i\frac{M_G}{r^2}\left(r\frac{d\theta}{d\tau}\right)\frac{dct}{d\tau}}-\frac{1}{r}\left(\frac{dr}{d\tau}\right)\left(r\sin\theta\frac{d\phi}{d\tau}\right)+\frac{\cot\theta}{r}\left(r\frac{d\theta}{d\tau}\right)\left(r\sin\theta\frac{d\phi}{d\tau}\right)\cdots\text{(the direction of latitude)}$$

where underlined parts are complex terms.

We expect that all informations of our main purposes are included in this system of equations.

§ 2. The Newton type equations of motion.

We consider the two-body problem concerned with the sun and the planet. Then the planet travels on the equator of the sun. Therefore, we put $\theta=\frac{\pi}{2}-i\Omega$ where Ω is a parameter relating to rotation.

Then we get a real coefficient equation, as follows;

The metric is $ds^2=-dct^2+dr^2+r^2(\cosh^2\Omega d\phi^2-d\Omega^2)$ and the system of equations is

$$(1)\quad \frac{d^2ct}{d\tau^2}=-\frac{M_G}{r^2}\frac{dr}{d\tau}\frac{dct}{d\tau}\cdots\text{(the direction of time),}$$

$$(2)\quad \frac{d^2r}{d\tau^2}=-\frac{M_G}{r^2}\left(\frac{dct}{d\tau}\right)^2+\frac{1}{r}\left\{\left(r\cosh\Omega\frac{d\phi}{d\tau}\right)^2-\left(r\frac{d\Omega}{d\tau}\right)^2\right\}\cdots\text{(the direction of radius),}$$

$$(3)\quad \frac{d}{d\tau}\left(r^2\frac{d\Omega}{d\tau}\right)=\left(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau}\right)\left(r^2\cosh\Omega\frac{d\phi}{d\tau}\right)\cdots\text{(the longitude areal velocity),}$$

$$(4)\quad \frac{d}{d\tau}\left(r^2\cosh\Omega\frac{d\phi}{d\tau}\right)=\left(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau}\right)\left(r^2\frac{d\Omega}{d\tau}\right)\cdots\text{(the latitude areal velocity).}$$

For holding a good discussion, we translate the above equations of Newton type into the equations of Kepler type.

Theorem 2 (The system of equations of Kepler type.)

The system of equations is

$$(1)'\quad \frac{dct}{d\tau}=C_0e^{\frac{M_G}{r}}\cdots\text{(the kinetic energy),}$$

$$(2)'\quad \frac{d^2}{d\tau^2}(r\sinh\Omega)=-\left(\frac{M_G}{r^2}\frac{dct}{d\tau}\right)\left(\tanh\Omega-r\cosh\Omega\frac{d\phi}{dct}\right)\cosh\Omega\left(\frac{dct}{d\tau}\right)\cdots\text{(the structure of space),}$$

$$(3)'\quad \left(r^2\frac{d\Phi}{d\tau}\right)^2=\left(r^2\cosh\Omega\frac{d\phi}{d\tau}\right)^2-\left(r^2\frac{d\Omega}{d\tau}\right)^2=C^2\cdots\text{(the law of equal areas),}$$

$$(4)'\quad r^2\cosh\Omega\frac{d\phi}{d\tau}=C\cosh\Theta'\ (\geqq 0),\ r^2\frac{d\Omega}{d\tau}=-C\sinh\Theta'$$

$$\Theta'=\int\left(\sinh\Omega\frac{d\phi}{dct}-\frac{M_G}{r^2}\right)dct\cdots\text{(the internal rotation).}$$

All the information in physics is contained in this system of equations.

We put the angular velocity $\frac{d\Phi}{d\tau}=\sqrt{\left(\cosh\Omega\frac{d\phi}{d\tau}\right)^2-\left(\frac{d\Omega}{d\tau}\right)^2}$, the orbit speed $r\frac{d\Phi}{d\tau}$

and the main equation $\left(\frac{d\frac{1}{r}}{d\Phi}\right)^2=\left(\frac{dr}{r^2d\Phi}\right)^2=-\frac{c^2}{C^2}+\frac{C_0^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\cdots(*)$.

We call C_0, C the energy coefficient and areas coefficient respectively.

Proof

(i) The kinetic energy.

From the equation $\frac{d^2ct}{d\tau^2}=-\frac{M_G}{r^2}\frac{dr}{d\tau}\frac{dct}{d\tau}\cdots(1)$.

$\left(\frac{dct}{d\tau}\right)^{-1}\frac{d^2ct}{d\tau^2}=-\frac{M_G}{r^2}\frac{dr}{d\tau}\quad\therefore\frac{d}{d\tau}\log\left(\frac{dct}{dc\tau}\right)=\frac{d}{d\tau}\left(\frac{M_G}{r}\right)$, therefore, we get the equation of the kinetic energy

$$\frac{dct}{d\tau}=\frac{c}{\sqrt{1-\left(\frac{v}{c}\right)^2}}=C_0e^{\frac{M_G}{r}}\cdots(1)'$$ where $e^{\frac{M_G}{r}}$ is a potential energy in the anti de-Sitter space and $\frac{M_G}{r}$ in its tangent space.

This equation $(1)'$ means the law of the conservation of energy. When we put the heights r_0, r and the speeds v_0, v at two points respectively, then

$$C_0=\frac{c}{\sqrt{1-\left(\frac{v_0}{c}\right)^2}}e^{-\frac{M_G}{r_0}}=\frac{c}{\sqrt{1-\left(\frac{v}{c}\right)^2}}e^{-\frac{M_G}{r}}.\quad \therefore\frac{\sqrt{1-\left(\frac{v_0}{c}\right)^2}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}=e^{-\frac{M_G}{r_0}+\frac{M_G}{r}}.$$

$$\therefore -\frac{1}{2}\left(\frac{v_0}{c}\right)^2+\frac{1}{2}\left(\frac{v}{c}\right)^2+\cdots=-\frac{M_G}{r_0}+\frac{M_G}{r}+\cdots,\ \text{where } M_G=\frac{GM}{c^2}.$$

This means the following conservation of energy

$$\frac{1}{2}mv^2-G\frac{mM}{r}=\frac{1}{2}mv_0^2-G\frac{mM}{r_0}=constant.$$

(ii) The law of equal areas.

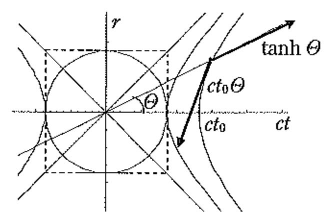
From the equation $(3)\times\left(r^2\frac{d\Omega}{d\tau}\right)-(4)\times\left(r^2\cosh\Omega\frac{d\phi}{d\tau}\right)$, we get

$$\left(r^2\frac{d\Phi}{d\tau}\right)^2=\left(r^2\cosh\frac{d\phi}{d\tau}\right)^2-\left(r^2\frac{d\Omega}{d\tau}\right)^2=C^2\cdots(3)'\ \text{(the law of equal areas)}.$$

(iii) The internal rotation.

From $(3)'$, we can put $r^2\cosh\Omega\frac{d\phi}{d\tau}=C\cosh\Theta'\ (\geqq 0)$, $r^2\frac{d\Omega}{d\tau}=-C\sinh\Theta'$.

then $\dfrac{-r^2\frac{d\Omega}{d\tau}}{r^2\cosh\Omega\frac{d\phi}{d\tau}}=\tanh\Theta'$ holds. (Cf. $\frac{dr}{dct}=\tanh\Theta$.)



The figure of an imaginary angle Θ

Therefore, from the equation (4)−(3), we get

$$\frac{d}{d\tau}(\sinh\Theta'+\cosh\Theta')=-\left(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau}\right)(\cosh\Theta'+\sinh\Theta').$$

$$\therefore\frac{de^{\Theta'}}{d\tau}=-\left(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau}\right)e^{\Theta'}\ \therefore\frac{d\Theta'}{d\tau}=\frac{d}{d\tau}\log e^{\Theta'}=-\left(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau}\right).$$

$$\therefore\Theta'=-\int\left(\frac{M_G}{r^2}\frac{dct}{d\tau}-\sinh\Omega\frac{d\phi}{d\tau}\right)d\tau\cdots(4)'\ \text{(the internal rotation)}.$$

(iv) The structure of space and the Minkowski's metric.

From the equations $(3)'$ and $(2)\times 2\left(\frac{dr}{d\tau}\right)-(1)\times 2\left(\frac{dct}{d\tau}\right)$, we get

$$\frac{d}{d\tau}\left(\frac{dct}{d\tau}\right)^2-\frac{d}{d\tau}\left(\frac{dr}{d\tau}\right)^2-\frac{d}{d\tau}\left(\frac{C^2}{r^2}\right)=0.$$

This means the Minkowski's metric in the time-space. Therefore,

$\left(\frac{dct}{d\tau}\right)^2-\left(\frac{dr}{d\tau}\right)^2-\frac{C^2}{r^2}=c^2$ holds. And then $\left(\frac{dr}{d\tau}\right)^2=-c^2+C_0^2e^{2\frac{M_G}{r}}-\frac{C^2}{r^2}$.

By this equation and the law of equal areas $r^2\frac{d\Phi}{d\tau}=C\cdots(3)'$, we get the main equation

$$\left(\frac{d\frac{1}{r}}{d\Phi}\right)^2=\left(\frac{dr}{r^2d\Phi}\right)^2=-\frac{c^2}{C^2}+\frac{C_0^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\ \text{in the tangent space.}$$

Moreover by the $(2)\times\sinh\Omega+(3)\times\frac{1}{r}\cosh\Omega$,

$$\frac{d^2r}{d\tau^2}\sinh\Omega+\frac{d}{d\tau}\left(r\frac{d\Omega}{d\tau}\right)\cosh\Omega+\frac{dr}{d\tau}\left(\frac{d\Omega}{d\tau}\right)\cosh\Omega$$

$$=-\left(\frac{M_G}{r^2}\frac{dct}{d\tau}\right)\left(\tanh\Omega-r\cosh\Omega\frac{d\phi}{dct}\right)\cosh\Omega\left(\frac{dct}{d\tau}\right)-\frac{1}{r}\left(r\frac{d\Omega}{d\tau}\right)^2\sinh\Omega\ \text{holds.}$$

Therefore, we get

$$\frac{d^2}{d\tau^2}(r\sinh\Omega)=\frac{d}{d\tau}\left\{\frac{d}{d\tau}(r\sinh\Omega)\right\}=\frac{d}{d\tau}\left\{\frac{dr}{d\tau}\sinh\Omega+r\cosh\Omega\frac{d\Omega}{d\tau}\right\},$$

$$=\frac{d^2r}{d\tau^2}\sinh\Omega+\frac{dr}{d\tau}\cosh\Omega\frac{d\Omega}{d\tau}+\sinh\Omega\frac{d\Omega}{d\tau}\left(r\frac{d\Omega}{d\tau}\right)+\frac{d}{d\tau}\left(r\frac{d\Omega}{d\tau}\right)\cosh\Omega,$$

$$=-\left(\frac{M_G}{r^2}\frac{dct}{d\tau}\right)\left(\tanh\Omega-r\cosh\Omega\frac{d\phi}{dct}\right)\cosh\Omega\left(\frac{dct}{d\tau}\right)\cdots(2)'\ \text{(the structure of space)}.$$

(A consideration of a special case)

This equation indicates a motion of free oscillation by Hooke's law, as follows;

$$\frac{d^2}{d\tau^2}\underline{\underline{(r\sinh\Omega)}}=-\frac{M_G}{r^3}\left(\frac{dct}{d\tau}\right)^2\left\{\underline{\underline{r\sinh\Omega}}-(r\cosh\Omega)^2\frac{d\phi}{dct}\right\}.$$

When the orbit is a circle and this oscillation is stable, the identity $\tanh\Omega\equiv r\cosh\Omega\frac{d\phi}{dct}$ holds, and this formula decides the value of parameter Ω.

We understand this parameter Ω is the rotating velocity.

From this identity and the relation $\frac{d}{d\tau}(r\sinh\Omega)=\frac{dr}{d\tau}\sinh\Omega+r\cosh\Omega\frac{d\Omega}{d\tau}$, the relations $\frac{d^2}{d\tau^2}(r\sinh\Omega)\equiv 0$, $\frac{d}{d\tau}(r\sinh\Omega)\equiv 0$ and $\frac{dr}{dct}\cdot r\cosh\Omega\frac{d\phi}{dct}\equiv -r\frac{d\Omega}{dct}$ hold.

The last equation yield two important equations, that is,

(i) Two parameters Θ, Θ' are equivalent, because $\frac{dr}{dct}\equiv -\frac{r\frac{d\Omega}{dct}}{r\cosh\Omega\frac{d\phi}{dct}}$.

(ii) The composition of energies represents not simply the sum but the product of two energies, because we put $\tanh\Omega_0 = r\cosh\Omega\frac{d\phi}{dct}$, then we get

$$\frac{dt}{d\tau}=\frac{1}{\sqrt{1-\left(\frac{dr}{dct}\right)^2-r^2\left\{\left(\cosh\Omega\frac{d\phi}{dct}\right)^2-\left(\frac{d\Omega}{dct}\right)^2\right\}}},$$

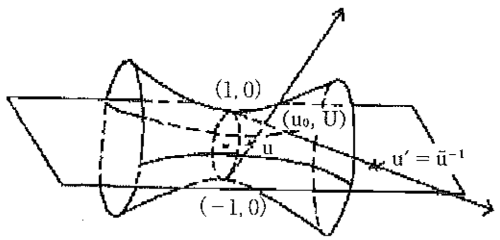
$$=\frac{1}{\sqrt{1-\left(\frac{dr}{dct}\right)^2}}\cdot\frac{1}{\sqrt{1-\left(r\cosh\Omega\frac{d\phi}{dct}\right)^2}}(=\cosh\Theta\cdot\cosh\Omega_0).$$

§ 3. The analysis and calculation of equations of motion.

We differentiate the kinetic energy (1) $\frac{dct}{d\tau}=C_0 e^{\frac{M_G}{r}}$ by the angle of rotation Φ, $d\Phi=\sqrt{(\cosh\Omega d\phi)^2-(d\Omega)^2}$.

Then we get the following relation to the potentials $e^{\frac{M_G}{r}}$ in the anti de-Sitter space and $\frac{M_G}{r}$ in its tangent space.

$$\frac{d}{d\Phi}\left(\frac{dct}{d\tau}\right)=\frac{d}{d\Phi}\left(C_0 e^{\frac{M_G}{r}}\right)=C_0\frac{e^{\frac{M_G}{r}}d\frac{M_G}{r}}{d\Phi}=C_0\frac{d\frac{M_G}{r}}{e^{-\frac{M_G}{r}}d\Phi}$$



The Anti de-Sitter space

Therefore, we get the main equation in the anti de-Sitter space, as follows;

$$\left(\frac{de^{\frac{M_G}{r}}}{d\Phi}\right)^2=\left(\frac{M_G d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)^2=M_G{}^2\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\right)e^{2\frac{M_G}{r}}\cdots(*)'.$$

The main equation in the above § 2 theorem is the relation of the potential in the tangent space.

We differentiate this main equation $(*)'$ by the angular parameter Φ, then

$$2\left(\frac{M_G d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)\frac{d}{d\Phi}\left(\frac{M_G d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)$$

$$=M_G{}^2\left[\left(2M_G\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{2}{r}\right)e^{2\frac{M_G}{r}}+\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\right)2M_G e^{2\frac{M_G}{r}}\right]\left(\frac{d\frac{1}{r}}{d\Phi}\right).$$

$$\therefore\frac{d}{e^{-\frac{M_G}{r}}d\Phi}\left(\frac{d\frac{1}{r}}{e^{-\frac{M_G}{r}}d\Phi}\right)=\left(M_G\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r}\right)e^{2\frac{M_G}{r}}+\left(-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}e^{2\frac{M_G}{r}}-\frac{1}{r^2}\right)M_G e^{2\frac{M_G}{r}},$$

$$=-\left(M_G\frac{c^2}{C^2}+\frac{1}{r}+M_G\frac{1}{r^2}\right)e^{2\frac{M_G}{r}}+2M_G\frac{C_0{}^2}{C^2}e^{4\frac{M_G}{r}},\ e^{2\frac{M_G}{r}}=1+2\frac{M_G}{r}+2\frac{M_G{}^2}{r^2}+\cdots,$$

$$\fallingdotseq\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)M_G+\left\{\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2-1\right\}\frac{1}{r}.$$

Therefore, we get the following differential equation,

$$\frac{d^2\frac{1}{r}}{\left(e^{-\frac{M_G}{r}}d\Phi\right)^2}=-\left\{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2\right\}\left[\frac{1}{r}-\frac{\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)M_G}{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}\right].$$

This solution is an ellipse and contains a circle, a parabola, and a hyperbola.

$$\frac{1}{r}=\frac{\left(2\frac{C_0{}^2}{C^2}-\frac{c^2}{C^2}\right)M_G}{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}\left[1+e\cos\left(\sqrt{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}\cdot e^{-\frac{M_G}{r}}\Phi\right)\right],$$

where "e" is a constant of integration ,i.e., an eccentricity.

(Cf 1.) The Schwarzschild metric $ds^2=-\left(1-\frac{2M_G}{r}\right)dct^2+\frac{1}{1-\frac{2M_G}{r}}dr^2+r^2(\sin^2\theta d\phi^2+d\theta^2)$

corresponds to the Minkowski's metric $ds^2=-d'ct^2+d'r^2+r^2(\sin^2\theta d\phi^2+d\theta^2)$

by the $d'ct=\sqrt{1-\frac{2M_G}{r}}dct$ and the $d'r=\frac{1}{\sqrt{1-\frac{2M_G}{r}}}dr$.

And the main equation in general theory of relativity is $\left(\frac{1}{\sqrt{1-\frac{2M_G}{r}}}\frac{d\frac{1}{r}}{d\phi}\right)^2=-\frac{c^2}{C^2}+\frac{C_0{}^2}{C^2}\frac{1}{1-\frac{2M_G}{r}}-\frac{1}{r^2}$.

And this is similar to the main equation $(*)$ in this paper, therefore we can expect the same results as Example 1 and Example 2 below.

Example 1 (The perihelion precession of Mercury)

We calculate the period of the Mercury orbit with respect to the angle $\varphi=e^{-\frac{M_G}{r}}\Phi$, as follows;

When Mercury travels around the sun, we get the period

$$\varphi=\frac{2\pi}{\sqrt{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}}\text{ from the relation }\sqrt{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}\cdot\varphi=2\pi.$$

Therefore, the advance of perihelion is $\frac{2\pi}{\sqrt{1-\left(8\frac{C_0{}^2}{C^2}-2\frac{c^2}{C^2}\right)M_G{}^2}}-2\pi\fallingdotseq 2\pi\left(\frac{4C_0{}^2-c^2}{C^2}\right)M_G{}^2$.

We substitute the concrete value for this formula, as follows;

the light speed is $c=2.99792458\times 10^8\mathrm{m}\cdot\mathrm{sec}^{-1}$, the Newton's constant of gravity is $G = 6.673 \times 10^{-11} m^3 \cdot kg^{-1} \cdot \sec^{-2}$,

the weight of the sun is $M = 1.989 \times 10^{30}$kg.

Therefore, we get $M_G = \frac{GM}{c^2} = 1476.55$m.

Moreover we put r_1 (a perihelion) is 4.60012×10^{10}m,

r_2 (an aphelion) is 6.98169×10^{10}m.

This two values are solutions of the equation $\left(\frac{dr}{d\tau}\right)^2 = -c^2 + C_0{}^2 e^{2\frac{M_G}{r}} - \frac{C^2}{r^2} = 0$.

Therefore, the values C_0 (the energy coefficient) and C (the areas coefficient) satisfy following simultaneous equations.

$$\frac{C^2}{r_1{}^2} - C_0{}^2 e^{2\frac{M_G}{r_1}} + c^2 = 0$$

$$\frac{C^2}{r_2{}^2} - C_0{}^2 e^{2\frac{M_G}{r_2}} + c^2 = 0,\ C_0,\ C > 0.$$

Then we get $C = 2.7129204353541553 \times 10^{15} m^2 \cdot \sec^{-1}$,

$C_0 = 2.997924541779737 \times 10^8 m \cdot \sec^{-1}$ ($\approx c$: the light speed).

Therefore, we get the value of the advance of perihelion, as follows;

$$360 \times 60\,(\text{min}) \times 60\,(\text{sec}) \times \left(\frac{4C_0{}^2 - c^2}{C^2}\right) M_G{}^2 \times 415\,(\text{round}) = 42.95626382458457''.$$

(Cf2.) In general theory of relativity, the value of the advance of perihelion is

$$2\pi \frac{3c^2}{C^2} M_G{}^2 = 2\pi \frac{3M_G}{r_m(1-e^2)}\,(\text{rad}) = 43.03''\ (\textit{per}\ 100\ \textit{years}).$$

Example 2 (The shift of light by the strong gravitational field.)

Generally, we put the speed v_1 at the perihelion r_1, then we devide the areas coefficient $r\frac{d\Phi}{d\tau} = \frac{r_1 v_1}{\sqrt{1-\left(\frac{v_1}{c}\right)^2}} = C$

by the energy coefficient $\frac{dct}{d\tau} = \frac{c}{\sqrt{1-\left(\frac{v_1}{c}\right)^2}} = C_0 e^{\frac{M_G}{r_1}}$, then we get the ratio $\frac{C}{C_0 e^{\frac{M_G}{r_1}}} = r_1 \frac{v_1}{c}$ of two coefficients.

We put the radius of the sun $R_\odot = 6.955 \times 10^8 m$ as the perihelion r_1 of light.

And we take the speed v_1 very close to the light speed. Then coefficient C and coefficient C_0 diverge to infinity. But the ratio of two coefficients converges, as follows;

when $v_1 \to c$ (light speed), $\frac{C}{C_0} = r_1 \frac{v_1}{c} e^{\frac{M_G}{r_1}} \to R_\odot e^{\frac{M_G}{R_\odot}} \fallingdotseq R_\odot$.

Therefore, the main equation converges to

$$\left(\frac{de^{\frac{M_G}{r}}}{d\Phi}\right)^2 = \left(\frac{M_G d\frac{1}{r}}{e^{-\frac{M_G}{r}} d\Phi}\right)^2 = M_G{}^2\left(\frac{1}{R_\odot{}^2} e^{2\frac{M_G}{r}} - \frac{1}{r^2}\right) e^{2\frac{M_G}{r}}.$$

And the orbit of the light is a hyperbola, as follows;

we put $\varphi = e^{-\frac{M_G}{r}}\Phi$, then $\frac{1}{r} = \frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}\left[1 + e\cos\left(\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi\right)\right]$.

(A) When the light is at the point $R_\odot = r_1$ (a perihelion),

we get $\frac{1}{R_\odot} = \frac{1}{r_1} = \frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}(1+e)$ from the relation $\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi = 0$.

Therefore, we get an eccentricity of the hyperbola, as follows;

$$e = \frac{1-\frac{8M_G{}^2}{R_\odot{}^2}}{\frac{2M_G}{R_\odot}} - 1 \fallingdotseq \frac{R_\odot}{2M_G} = 2.35515 \times 10^5 \text{ where } \frac{M_G}{R_\odot} \text{ is } 2.12301 \times 10^{-6}.$$

(B) When the light is at the point r_2 (an aphelion),

we get $\frac{1}{r_2} = \frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}(1-e)$ from the relation $\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi = \pi$.

Therefore, we get a value of the aphelion, as follows;

$$r_2 = \frac{1-\frac{8M_G{}^2}{R_\odot{}^2}}{\frac{2M_G}{R_\odot{}^2}(1-e)} \fallingdotseq \frac{R_\odot}{2M_G(1-e)} R_\odot = \frac{e}{1-e} R_\odot = -6.95503 \times 10^8\ (< -R_\odot)$$

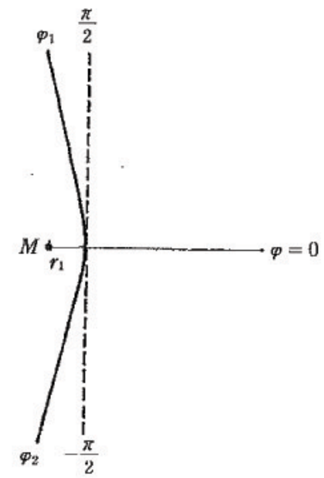
(C) When the light is at the point $r \to \infty$ (an asymptote),

we get $0 = \left(\frac{1}{\infty} =\right) \frac{\frac{2M_G}{R_\odot{}^2}}{1-\frac{8M_G{}^2}{R_\odot{}^2}}\left[1 + e\cos\left(\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi\right)\right]$ from the relation

$$\frac{\pi}{2} < \sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi < \pi.$$

$$\therefore \cos\left(\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi\right) = -\frac{1}{e} < 0 \ \therefore \sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi - \frac{\pi}{2} \fallingdotseq \frac{1}{e}$$

$$\therefore \sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}} \cdot \varphi \fallingdotseq \pm\left(\frac{\pi}{2} + \frac{1}{e}\right) \therefore \varphi \fallingdotseq \pm \frac{\frac{\pi}{2} + \frac{1}{e}}{\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}}$$



The shift of light

Therefore, we get a shift angle of light, that is,

$$\Delta\varphi = 2\frac{\frac{\pi}{2}+\frac{1}{e}}{\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}}-\pi = \left(\frac{\pi}{\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}}-\pi\right)+\frac{\frac{2}{e}}{\sqrt{1-\frac{8M_G{}^2}{R_\odot{}^2}}},\ \frac{1}{e} \fallingdotseq \frac{2M_G}{R_\odot}$$

$$\fallingdotseq \frac{4M_G}{R_\odot} = 8.49204\times 10^{-6}\,(\mathrm{rad}) = 1.75161''$$

(Cf 3.) In general theory of relativity, the Schwarzschild metric of the light is

$$-d'ct^2 + d'r^2 + r^2(\sin^2\theta d\phi^2 + d\theta^2) = 0,$$

where the $d'ct = \sqrt{1-\frac{2M_G}{r}}dct$ and the $d'r = \frac{1}{\sqrt{1-\frac{2M_G}{r}}}dr$.

And the shift angle is $4M_G\frac{C_0}{C} = \frac{4M_G}{R_\odot} = \frac{4\times 1476.55}{6.955\times 10^8} = 8.5\times 10^{-6}\,(\mathrm{rad}) \fallingdotseq 1.75''$.

References

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(2) Y. Takemoto, The Equation of Gravitational Force and the Electromagnetic Force, Bull. of NBU Vol. 36, No. 2 (2008-Mar.) pp. 14-22.